Subsidies, Patent and International Technology Diffusion in North-South Duopoly[1]

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Abstract

This paper analyzes the relationship between public policy instruments and technology diffusion in a North-South duopoly within an inter-temporal model where time is continuous. Initially, the Northern firm benefits from a new technology while the Southern firm only uses an old technology. There is a monopoly length of the new technology for the Northern firm. At the end, there is technology diffusion from the Northern firm to the Southern firm. The Northern firm implements patent in order to slow down diffusion. This article studies the impact of the Northern country's production subsidy, the Northern country's patent subsidy and the Southern country's production subsidy. The results show that the Northern government's subsidies slow down technology diffusion by increasing the monopoly length of the new technology while the Southern government's subsidy generally accelerates it. Welfare analysis demonstrates that the Northern government is encouraged to subsidize production and to tax expenditures on patent. The results hold both under Cournot and Bertrand competition. The Southern is encouraged to subsidize production under Cournot competition. But under Bertrand competition, production tax is optimal.

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Keywords: technology diffusion, patent, production subsidy, patent subsidy.

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1. Introduction

The study of international technology diffusion has become an important aspect of economics. Technology diffusion is a central topic for developing countries because it corresponds to a development issue. It represents a way to improve industries which enhance economic development, especially capital intensive industries. As a matter of fact, modern technology in industries increases workers' wage, purchasing power and standard of living for developing countries. It improves working conditions. Technology diffusion on process may also involve product innovations which benefit from developing countries' consumers. Furthermore, there is now much evidence to support the assumption that public policy instruments like subsidy have an impact on diffusion. But determining the sign of such impact is complex.

Technology diffusion occurs because of information diffusion (Geroski, 2000). With modern telecommunications, information is easily available everywhere. It creates technological spillovers. Keller (2002) says that *"telecommunications and the Internet ensure that people in all countries have access to the same pool of technological endowment* [p. 120]*."* As a matter of fact, technology diffusion is not something avoidable. Technology may be diffused from developed countries to other developed countries. But it may also be diffused from developed countries to developing countries. Developing countries acquire modern production process that developed countries previously discover by innovating. Note that developing countries' innovations investments are low. According to Keller (2004), *"only a handful of rich countries account for most of the world's creation of new technology. The pattern of worldwide technical change is thus determined in large part by international technology diffusion* [p. 752]*."* Then, developing countries' technological endowment depends on technology diffusion.

The issue of technology diffusion has been studied in the economic literature. Note that Keller (2004) makes a wide survey of economic literature on such topic. A first factor of technology diffusion is R&D investment. There are R&D spillovers to foreign countries because human capital involves spillovers especially when foreign countries also invest in R&D. But we have to find the factor of such spillovers. Generally, the speed of technology diffusion between two countries increases with the volume of trade between them, especially the trade of intermediate goods (Rivera-Batiz and Romer, 1991; Grossman and Helpman, 1991; Eaton and Kortum, 2002). Developing countries import intermediate goods from developed countries and must use modern technology to produce final goods. Generally, international trade involves information flows between countries because of business interactions. Coe and Helpman (1995) find close results empirically by showing that R&D spillovers effect increases with international trade openness. Economic literature also proved that the speed of technology diffusion between two countries increases with the stock of Foreign Direct Investments (FDI) is high because of FDI spillovers for the country which receives it (Griliches and Hausman, 1986; Keller, 2002; Griffith, Redding and Simpson, 2003). For example, Japanese's R&D investments in Asian developing economies were multiplied by nine from 1993 to 2007 (UNCTAD, 2011). Developing countries like China, Korea or India benefited from such Japanese investments. Multinational Firms (MNF) use labor force from a foreign country where they are located. Spillovers between countries appear because of workers training by MNF (Aitken and Harrisson, 1999; Fosfuri, Motta and Rønde, 2001). This is the reason why the relationship between MNF and subsidiaries located in foreign countries is clearly overriding with respect to FDI (Markusen, 2002). International trade and FDI represent two important factors of technology diffusion. Ethier and Markusen (1996) studies the choice between international trade and FDI for a firm which wants to sell its product in a foreign country by using a theoretical dynamic framework with technological externalities. A firm from the domestic country discovers a new product and benefits from a temporary monopoly (which equals to two periods in the model). The firms from the foreign country do not invest in research in order to discover new products. The firm has a choice between to export from its domestic country or to locate a part of its output in the foreign country. According to authors, localization involves stronger information absorption for other firms in the foreign country. Other firms from the foreign country can produce the new product faster. Then, technology diffusion seems to be faster with localization than with exports. But it represents a cost for the domestic firm because there is no longer monopoly on the new product. The choice between export and localization depends on transport cost of export and monopoly rent of localization.

Economic literature analyzes the impact of public policies (especially trade policies) on technology diffusion. The reference model without diffusion is the framework of Spencer and Brander (1983) which analyzes the positive effect of R&D subsidy on domestic R&D investment and national welfare in an international duopoly. Cheng (1987) designs a close framework within a dynamic model. Considering international technology diffusion, he shows that the R&D subsidy which only satisfies domestic interest may benefit from the foreign firm. It may also enhance diffusion. Grossman and Helpman (1991) implement a theoretical macroeconomic model with technological spillovers and study the economic impact of trade openness of a small country. They show that trade policies which reduce (respectively promote) international trade, especially trade of intermediate goods, like tariffs or quotas (respectively subsidies) have a negative (respectively positive) effect on innovations and technology diffusion via knowledge spillovers. For example, a tariff cut involves an increase of trade volume, of trade by variety of intermediate goods and of stock of human capital by variety. Then, spillovers to foreign countries are stronger. Miyagiwa and Ohno (1997) design an interesting structure to analyze the impact of subsidies on R&D and welfare. They use a dynamic theoretical model. Initially, two firms use an old technology. At each point of time, they invest in R&D to discover a new technology. There is a probability to discover it at each point of time for each firm. When a firm discovers the new technology, it no longer invests in R&D. But the other one still invests until it discovers the new technology too. The first one benefits from an exogenous monopoly period of the new technology which corresponds to the speed of diffusion. Reppelin-Hill (1999) makes an econometric study of the relationship between trade openness and speed of clean technology diffusion by using the example of steel industry. He demonstrates that diffusion of clean technology is *"faster in countries that have more open trade policy regimes* [p. 284]*."* Geroski (2000) makes a survey of factors of technology diffusion. Information diffusion involves technology diffusion. He suggests that governments can subsidize technological externalities *"to promote ... communication … and to motivate them* [p. 621]*."* Battisti (2008) also uses the example of environment and establishes that technology diffusion is a low process. Governments' policies may increase R&D investment but *"should also look at the adoption and the extent of use of innovations because that is the place where the generation of the benefits from inventions takes place* [p. 528]*."*

Economic literature also studies the impact of other variables which influence the speed of technology diffusion. Geographic distance between two countries has an impact on technology diffusion because of its effect on bilateral trade. Generally, previous studies proved that technology diffusion is faster inside a country than between two countries (Jaffe, Trajtenberg and Henderson, 1993; Branstetter, 2001; Eaton and Kortum, 1999). There is a border effect. Nevertheless, Irwin and Klenow (1994) do not find that the speeds of technology diffusion are not significantly different by taking the example of US firms as compared to foreign firms in semi-conductors industry from 1974 to 1992. Other papers study the significant negative effect of the distance in kilometers on the speed of technology diffusion (Keller, 2002; Bottazi and Peri, 2003).

Furthermore, there is a strong correlation between technology diffusion and industrial protection. For example, patent has an effect on diffusion because it represents a way to protect information and technological endowment for firms. Patent may slow down technology diffusion by giving monopoly on product or process to firms from developed countries. However, citing patents may be a measure of technology diffusion. Eaton and Kortum (1999) consider that patent filing in foreign country represents another measure of technology diffusion (even if it is not a perfect measure). They prove that diffusion depends on the possibility of patent filing in the foreign country. But it decreases with the patent filing cost that the firm must pay. Pakes (1996) also mentions the role of patent cost when agents want to keep the patent in force. Economic literature also designed models with patent by studying the impact on national welfare. Such patent may involve monopoly rent for firm which publishes it. Authors establish that patent encourages innovation because firm must discover new process or new product before patent publication. A first part of literature considers that imitation of competitors is never a threat and identifies that optimal patent length is finite by considering that policy makers selects patent length (Nordhaus, 1969; Scherer, 1972). Another part considers that imitation is costless, and introduces patent length and breadth (Tandon, 1982; Klemperer, 1990; Gilbert and Shapiro, 1990). According to Gilbert and Shapiro (1990), patent breadth cannot be clearly defined: *"a broader patent allows the innovator to earn a higher flow rate of profit during the lifetime of the patent* [p. 107]*."* Even if patent breadth leads deadweight loss, they demonstrate that optimal patent length is infinite because it minimizes social costs. Gallini (1992) introduces endogenous imitation cost by considering *"the ability of competitors to invent around* [p. 52]*."*He finds that optimal patent length is short in order to avoid imitation. Such result contrasts with previous studies. Mathew and Mukherjee (2014) study the impact of patent regime on FDI inwards in a North-South structure. The incentive of Northern firms' FDI in the Southern country increases with a strong patent regime especially when the costs of Southern innovation are high. A strong patent regime corresponds to a situation where only the patent holder can sell its product. Here, the impact of patent regime on technology diffusion is ambiguous because it may increase FDI inwards in the Southern country but it may also involve longer monopoly for Northern firms on sales of new products.

The objective of this paper is to evaluate the impact of subsidies on technology diffusion using a simple theoretical framework. We also search the impact on other strategic variables like profits, consumer surplus or national welfares. We use a framework which is close to Miyagiwa and Ohno (1999). But it is a North-South duopoly. We consider a duopoly with one firm of a Northern country and one firm of a Southern country. The Northern firm produces one part of its output in its domestic country and the other one in the Southern country. Then, we consider that the Northern firm relocates one part of its output in the Southern country in order to benefit from lower production cost. For example, we could consider that the firm produces intermediate goods in the Southern country and export such goods to the Northern country to produce finite good because of the ability of qualified workers in developed country. A good example of such structure is Maquiladoras in Mexico. US firms locate their output in Mexico to benefit from lower production cost and lower taxes and re-export it in United States. Another example is automobile industry in Germany where firms import intermediate goods from Eastern Europe and produce finite good in Germany. In our framework, note that we still denote such firm as the "Northern firm" because it keeps its headquarters in the Northern country. The Southern firm produces all its output in the Southern country. Here, technology diffusion occurs with the Northern firm's location in the Southern country.

Another difference with Miyagiwa and Ohno (1999) is that, initially, a firm of a Northern country already benefits from the new technology while another firm of a Southern country uses the old technology. Developed countries benefits from larger capital endowment while developing countries benefits from larger labor endowment. Here, technology diffusion does not correspond to technology transfer from the Northern country to the Southern country because the Northern firm locates its output in the Southern country. It corresponds to transfer from the Northern firm to the Southern firm. The new technology diffusion is endogenous. We study the case where the Northern firm publishes patent in the Southern country to increase the monopoly length of the new technology. Patent represents a way to protect the Northern firm's information to slow down technology diffusion. The Southern firm can use the new technology only at the term of patent. In such North-South framework, we consider that the Northern country's patent always slows down technology diffusion to the Southern country. We omit the possibility of patent citations. Nevertheless, it corresponds to a situation where patent involves disputes between firms or countries. For example, The New York Times published an article in 2012 about technology war between Google and Apple by mentioning that patent is a way to prevent a firm to be competitive on a specific market and that a firm often sues another one because of patent infringements.[3] We also consider a simple case where patent only involves process protection. But technological endowment asymmetry between the Northern firm and the Southern firm already exists. The Northern firm previously invested to discover the new technology to be more competitive. Then, unlike previous studies, we disregard the fact that patent could encourage inventions or innovations. Furthermore, we take into account that the Northern firm faces a patent filing cost. Papers which study the optimal patent length generally do not introduce it. Papers which introduce patent cost study the effect of patent as compared to a situation without patent but do not study the optimal patent length. Here, we try to implement a new structure by implementing both patent length and patent filing cost. Such cost increases with the term of patent because the Northern firm has to pay maintenance fees to keep the patent in force. Then, the Northern firm selects expenditures on patent by choosing the term of patent. According to TRIPs Agreements (Article 33), *"The term of protection available shall not end before the expiration of a period of twenty years counted from the filing date."* Nevertheless, there are some ways to choose a period which is different than twenty years, especially utility model which has fewer requirements. Utility model is one particular type of patent which is less stringent in terms of protection length and filing cost. Generally, the term of utility model is around ten years. Note that the Northern firm's patent only slows down diffusion because technology diffusion is not avoidable (Keller, 2002). Then, infinite patent length is not possible.

[3] See the newspaper article of The New York Times (October 7, 2012) by Charles Duhigg and Steve Lohr entitled "In technology wars, using the patent as a sword."

We analyze the effect of three policy instruments on technology diffusion across the impact on the Northern firm's expenditures on patent: a production subsidy and a patent subsidy implemented by the Northern government, and a production subsidy implemented by the Southern government. We focus on "behind-the-border" policies rather than "at-the-border" policies like imports tariffs or quantitative restrictions. Currently, governments use "behind-the-border" policies more frequently for three reasons: (i) it represents a way to benefit domestic firms over foreign firms; (ii) it escapes the notice of the World Trade Organization (WTO) which is more effective in prohibiting instruments which represent a direct barrier on international trade flows; (iii) according to WTO, import tariffs are bound and cannot be increased above a certain level. Evenett (2013) describes the rise of trade policy instruments as the *"protectionism's quiet return"* and mentions that *"non-traditional forms of protections still dominate crisis-era protectionism* [p. 28]*."* Traditional forms like tariffs only represent less than 40% of the protectionism measures implemented since 2008. Production subsidy is classic trade policy. WTO tries to prevent production subsidies because it may create trade distortions. However, governments can easily subsidize indirectly the domestic firm's production across tax cuts or employment subsidies. The Southern government's subsidy is a way to find whether a trade policy implemented by the Southern government accelerates technology diffusion. The Northern government can also subsidize its domestic firm's expenditures on patent. It corresponds to patent subsidy. In 2002, the Belgium government implemented subsidies for small and medium-sized businesses *"to register and to maintain a patent."* In 2010, the subsidy rate achieved 70 percent of the patent filing cost (Source: Europa). Munari and Xu (2011) make an overview of ten experiences of patent subsidies over the 2000s: *"the use of patent subsidies, in particular in favor of SMEs, has recently gained an increased attention by policy-makers* [p. 5]*."*

We consider two competition modes: Cournot and Bertrand. Under Cournot competition, firms select optimal level of output while, under Bertrand competition, they select optimal level of price. In various studies, there is a lack of robustness with respect to the competition mode. For example when the domestic government wants to implement a trade policy on the domestic exports, under Cournot competition, the optimal policy is an export subsidy (Brander and Spencer, 1985). But under Bertrand competition, it is an export tax (Eaton and Grossman, 1986).

The results of this paper show that the Northern government's production subsidy and patent subsidy slow down technology diffusion between the two firms by increasing the monopoly length of the new technology. Then, policy instruments implemented by developed countries have negative effect on technology diffusion. But the patent subsidy decreases the Northern country's national welfare. In that case, the Northern government is encouraged to tax its domestic firm's expenditures on patent. The production subsidy always increases the Northern country's national welfare until an optimal level. Moreover, the Southern government's production subsidy generally accelerates technology diffusion by reducing the monopoly length of the new technology. But under Bertrand competition, the Southern government is not encouraged to subsidize its domestic firm's production. Otherwise, these results generally hold both under Cournot and Bertrand competition.

Section 2 introduces the model under Cournot competition with patent and Section 3 analyzes the impact of the Northern government's production and patent subsidies and the impact of the Southern government's production subsidy. Section 4 verifies whether results hold under Bertrand competition. Section 5 discusses the results and Section 6 concludes.

2. The model under Cournot competition

Consider a duopoly with a firm of a Northern country and another of a Southern country. As we said in Introduction, the Northern firm produces one part of its output in the Northern country and the other one in the Southern country because of a lower labor cost. Consider for example that the Northern firm produces intermediate goods in the Southern country and exports them to the Northern country to produce finite goods. The Southern firm produces all its output in its domestic country. There is competition on the Northern market. Each firm sells its output in the Northern country. Such assumption permits to study a simple case where firms compete on a market of a developed country by considering a larger market size with respect to developing country.

Suppose a dynamic model where time is continuous and defined over $[0, \infty)$. Initially, the Northern firm benefits from a new technology denoted by $\overline{\mu}$ while the Southern only uses an old technology denoted by $\underline{\mu}^*$. Here, technology diffusion is defined by technology transfer from the Northern firm to the Southern firm. The new technology diffusion occurs at period T over $[0, \infty)$. The Southern firm can use the new technology $\overline{\mu}^*$ from such point of time. The Northern firm benefits from $\overline{\mu}$ over $[0, \infty)$. Here, we consider that the new technology diffusion occurs because of information diffusion with telecommunications like the Internet and because of the Northern firm's location in the Southern country.

In the body of this paper, we use a general form of this model. In the Appendix, we use three specific demand functions and an iso-elastic function for the monopoly length of the new technology (see Appendix A).

Assumption 1: The Northern (respectively Southern) firm's marginal cost is constant and denoted by c (respectively c^*). Each firm's technological endowment influences its marginal cost: $c = c(\mu)$, with $\mu = \overline{\mu}$, $c^* = c^*(\mu^*)$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. We have: $c^*\left(\underline{\mu}^*\right) > c^*\left(\overline{\mu}^*\right)$. The value of c^* changes at time T.

Consider a two stages model. At each point of time, both firms select the level of output which maximizes its static profit flows. The Northern firm publishes patent in the Southern country to increase the monopoly length of the new technology. The patent filing cost increase with the patent length. Then, taking into account previous results, the Northern firm selects its expenditures on patent (i.e. the patent length) which maximizes its discounted sum of profit flows.

The Northern (Southern) firm produces x (y) at each point of time over $[0, \infty)$. Both firms sell a homogenous good only in the Northern market.

Assumption 2a: At each point of time, there is a Cournot competition on the Northern market. Each firm sells a homogenous good. We denote p the market price with the inverse demand function: $p(X) = p(x + y)$, where X denotes the total supply. The market price decreases with the total supply: $p' = \mathrm{d}p/\mathrm{d}X < 0$.

At each point of time, the static profit flows are:

$$\pi(x, y) = xp(x + y) - c(\overline{\mu})x - F \quad (1)$$

$$\pi^*(x, y) = yp(x + y) - c^*(\mu^*)y - F^*, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (2)$$

Consider linear total cost functions where F (F^*) denotes the Northern (Southern) firm's fixed cost. Each firm selects the level of output which maximizes its profit. The first order conditions give the following reaction functions:

$$x(y) = [c(\overline{\mu}) - p(x + y)]/p', y(x) = [c^*(\mu^*) - p(x + y)]/p' \quad (3)$$

Each profit decreases with the foreign output: $\pi_y = xp' < 0, \pi_x^* = yp' < 0$.[4]

Assumption 3a: The second order conditions are verified: $\pi_{xx} = xp'' + 2p' < 0$, $\pi_{yy}^* == yp'' + 2p' < 0$. Cross effects are also negative: $\pi_{xy} = xp'' + p' < 0$, $\pi_{yx}^* = yp'' + p' < 0$. Own effects are stronger than cross effects: $|\pi_{xx}| > |\pi_{xy}|$, $|\pi_{yy}^*| > |\pi_{yx}^*|$.

Stability condition is verified: $D = \pi_{xx}\pi_{yy}^* - \pi_{xy}\pi_{yx}^* > 0$.

We search the effect of c^* on outputs, price and profits to determine the effect of the Southern firm's technological endowment. Differentiating the first order conditions $\pi_x = 0$ and $\pi_y^* = 0$ with respect to x, y and c^*, we have: $\mathrm{d}x/\mathrm{d}c^* = -\pi_{xy}/D > 0$, $\mathrm{d}y/\mathrm{d}c^* = \pi_{xx}/D < 0$, $\mathrm{d}X/\mathrm{d}c^* = (\pi_{xx} - \pi_{xy})/D < 0$. The Southern firm's marginal cost decreases (increases) the Southern (Northern) firm's output. It decreases the total supply. It increases the market price: $\mathrm{d}p/\mathrm{d}c^* = p'\,\mathrm{d}X/\mathrm{d}c^* > 0$. Therefore, it decreases the Northern country's consumer surplus by increasing p and reducing X.

[4] We use subscripts to denote partial derivatives.

Let us search the effect on profits by using the maximum profits. We denote $\hat{\pi}$ and $\hat{\pi}^*$ such profits which solve the following programs:

$$\hat{\pi}(c^*) = \max_{x>0} \pi(x, y) \text{ subject to } \pi_y^* = 0, \text{ i.e. } V(c^*) = \pi[\hat{x}(c^*)] \quad (4)$$

$$\hat{\pi}^*(c^*) = \max_{y>0} \pi^*(x, y) \text{ subject to } \pi_x = 0, \text{ i.e. } V^*(c^*) = \pi^*[\hat{y}(c^*)] \quad (5)$$

$\hat{x}$ ($\hat{y}$) denotes the Northern (Southern) firm's output which permits to achieve the maximum profit, i.e. the level of output which maximizes its profit: $\hat{x}(c^*) = \text{argmax}_{x>0}\, \pi(x, y)$, $\hat{y}(c^*) = \text{argmax}_{y>0}\, \pi^*(x, y)$. The Envelope Theorem gives:

$$\frac{\mathrm{d}\hat{\pi}(c^*)}{\mathrm{d}c^*} = \frac{\partial \pi(x,y)}{\partial c^*} - \lambda \frac{\partial \pi_y^*}{\partial c^*} = \lambda > 0 \quad (6)$$

$$\frac{\mathrm{d}\hat{\pi}^*(c^*)}{\mathrm{d}c^*} = \frac{\partial \pi^*(x,y,c^*)}{\partial c^*} - \lambda^* \frac{\partial \pi_x}{\partial c^*} = -\hat{y}(c^*) < 0 \quad (7)$$

The parameters λ and λ^* are Lagrange multipliers: $\lambda, \lambda^* > 0$. Therefore, the Southern firm's marginal cost decreases (increases) the Southern (Northern) firm's profit.

According to previous results, outputs, price and profits depend on the Southern firm's technological endowment. The Northern (Southern) firm's output and the Northern (Southern) firm's profit flows are stronger (lower) when the Southern firm uses the old technology: $x\left(\underline{\mu}^*\right) > x(\overline{\mu}^*)$, $y\left(\underline{\mu}^*\right) < y(\overline{\mu}^*)$, $\pi\left(\underline{\mu}^*\right) > \pi(\overline{\mu}^*)$, $\pi^*\left(\underline{\mu}^*\right) < \pi^*(\overline{\mu}^*)$. The total supply is lower and the market price is stronger: $X\left(\underline{\mu}^*\right) < X(\overline{\mu}^*)$, $p\left(\underline{\mu}^*\right) > p(\overline{\mu}^*)$. Then, the consumer surplus is lower.

The Northern firm benefits from a monopoly of the new technology until T and publishes a patent at time 0 in order to increase such monopoly. The period $[0, T)$ is called the monopoly length. The patent filing leads a filing cost which increases with such patent length. We denote k the Northern firm's expenditures which are necessary to implement the patent.

Assumption 4a: The Northern firm's monopoly length T of the new technology depends on the expenditures on patent k: $T = T(k)$. T increases with k: $T'(k) = \mathrm{d}T(k)/\mathrm{d}k > 0$. T increases with k: $T'(k) = \mathrm{d}T(k)/\mathrm{d}k > 0$. Furthermore, we have: $T''(k) = \mathrm{d}^2T(k)/\mathrm{d}k^2 \leq 0$.

Note that it seems intuitive to consider a linear relationship between the monopoly length T and the expenditures on patent k i.e. $T'' = 0$. Nevertheless, we can also consider $T'' < 0$ because the Southern firm may benefit from diffusion of alternative modern technology which is not patented with time. On the other hand, it seems complex to consider $T'' > 0$. Hence, we have: $T'' \leq 0$.

Assumption 5: The Northern firm is encouraged to publish the patent: $\pi\left(\underline{\mu}^*\right) - k > \pi(\overline{\mu}^*)$.

Taking into account the previous results, the Northern firm maximizes a discounted sum of profit flows, denoted by Π, with respect to k:

$$\max_{k\geq 0} \Pi(k) = \int_0^{T(k)} e^{-i\tau}\hat{\pi}\left(\underline{\mu}^*\right)\mathrm{d}\tau + \int_{T(k)}^{\infty} e^{-i\tau}\hat{\pi}\left(\overline{\mu}^*\right)\mathrm{d}\tau - k \quad (8)$$

where i denotes the interest rate which discounts flows. Time is denoted by τ. We use exponentials because of continuous time. Integrating, we have:

$$\max_{k\geq 0} \Pi(k) = \frac{\left[1-e^{-iT(k)}\right]\hat{\pi}\left(\underline{\mu}^*\right)+e^{-iT(k)}\hat{\pi}\left(\overline{\mu}^*\right)}{i} - k \quad (9)$$

The first order condition $\Pi_k = 0$ involves:

$$\frac{1}{T'(k)e^{-iT(k)}} = \hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right) \quad (10)$$

The second order condition is verified: $\Pi_{kk} = [T''(k) - iT'(k)^2]e^{-iT(k)}\left[\hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right)\right] < 0$. We denote $K(k)$ the left side of the previous equation. We have: $K(k) = \hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right)$. The function K increases with k:

$$K'(k) = \frac{iT'(k)^2 - T''(k)}{T'(k)^2 e^{-iT(k)}} > 0 \quad (11)$$

A simple interpretation of (11) stems for rewriting as: $k = \psi\left[\hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right)\right]$ with $\partial\psi/\partial\left[\hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right)\right] > 0$. Hence, k increases with the difference in profit $\left[\hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}\left(\overline{\mu}^*\right)\right]$.

3. The impact of public policy instruments

Let us study the impact of public policy instruments. The Northern government can subsidize its Northern firm's production or the Northern firm's expenditures on patent while the Southern government can only subsidize its domestic firm's production.

Assumption 6: Each production subsidy is implemented over $[0, \infty)$. But the Northern government's patent subsidy is only implemented at time 0.

Even if its Northern firm locates its output in the Southern country, the Northern government may be encouraged to implement public policy instruments because of political reasons. The Northern government tries to implement an instrument which increases the Northern country's national welfare as compared to free trade. An instrument which increases the consumer surplus is politically desirable. Furthermore, we can also justify it by employment reasons in the Northern country even if the Northern firm produces one part of its output in the Southern country. A subsidy from the

Northern government represents a way to hire further employees in the Northern country. We are going to make welfare analysis to find whether governments are encouraged to implement such subsidies.

3.1. The Northern government's production subsidy

The Northern government subsidizes its domestic firm's production. We denote s the production subsidy. The Southern firm's profit expression remains the same as Equation (2). The Northern firm 's profit expression becomes:

$$\pi(x, y, s) = xp(x+y) - c(\overline{\mu})x + sx - F \tag{12}$$

The Northern firm's reaction function is: $x(y) = [c(\overline{\mu}) - p(x+y) - s]/p'$. The Southern firm's reaction function remains the same as under free trade. The second order conditions and cross effects also remains the same. Let us find the effect of the subsidy on outputs, price and profits. We can determine it easily because we already study the impact of the Southern firm's marginal cost. According to (12), the production subsidy s has the same impact on profit as a negative marginal cost. The impact of c is symmetrical to the impact of c^*. Then, the subsidy s increases (decreases) the Northern (Southern) firm's output, increases the total supply, and decreases the market price by decreasing the Northern firm's total cost. Therefore, it increases consumer surplus. Finally, it increases (decreases) the Northern (Southern) firm's profit. Note that the positive effect of the subsidy on the Northern firm's profit equals its equilibrium level of output. Using the maximum profits $\hat{\pi}$ and $\hat{\pi}^*$, the Envelope Theorem gives:

$$\frac{\mathrm{d}\hat{\pi}(\mu^*,s)}{\mathrm{d}s} = \frac{\partial \pi(x,y,s)}{\partial s} - \lambda \frac{\partial \pi_y^*}{\partial s} = \hat{x}(\mu^*, s) > 0, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \tag{13}$$

Now, we search the effect of the subsidy s on technology diffusion. We search the effect on k. The Northern firm selects the expenditures on patent (i.e. the patent length) which maximizes its discounted sum of profit flows. The first order condition involves the same expression as under free trade except that the profit flows now depend on the production subsidy s.

Proposition 1: The Northern government's production subsidy increases the Northern firm's expenditures on patent k and the monopoly length T. Therefore, it slows down the new technology diffusion from the North to the South. Furthermore, the subsidy which maximizes the Northern country's national welfare is positive. The Northern government is encouraged to subsidize its domestic firm's production.

Proof: The Northern firm's expenditures on patent k depends on the difference in profit $\left[\hat{\pi}\left(\underline{\mu}^*, s\right) - \hat{\pi}\left(\overline{\mu}^*, s\right)\right]$. An increase of such difference leads an increase of k. The production subsidy increases k if the gain of Northern firm's profit flows is stronger when the Southern firm uses the old technology i.e. $\hat{\pi}_s\left(\underline{\mu}^*, s\right) > \hat{\pi}_s(\overline{\mu}^*, s)$. The Northern firm's output $\hat{x}(\mu^*, s)$ represents the marginal profit gain with the subsidy s; see Equation (13). Previously, we proved that the Southern firm's marginal cost c^* increases the Northern firm's output. Therefore: $\hat{x}\left(\underline{\mu}^*, s\right) > \hat{x}(\overline{\mu}^*, s)$, because $c^*\left(\underline{\mu}^*\right) > c^*(\overline{\mu}^*)$. We have now: $\hat{\pi}_s\left(\underline{\mu}^*, s\right) > \hat{\pi}_s(\overline{\mu}^*, s)$. The positive effect of s on the Northern firm's profit flows is stronger when the Southern firm uses the old technology. The production subsidy increases the difference in profit. Hence: $\mathrm{d}k/\mathrm{d}s > 0$. Then, the monopoly length T increases as compared to free trade and the new technology diffusion slows down.

The Northern government's production subsidy increases k because the positive effect on its profit is stronger when the Southern firm uses the old technology (its output is greater). It explains why the Northern firm is encouraged to increase the expenditures on patent in order to benefit from a longer monopoly of the new technology. The subsidy slows down technology diffusion.

We can summarize the economic impact of s (see Appendix B). It increases (decreases) the Northern (Southern) firm's discounted sum of profits. It reduces the market price but it enhances the total supply: the Northern country's consumer surplus increases as compared to free trade. It represents government expenditures: it reduces the Northern country's public revenues. The total effect on the Northern (Southern) country's national welfare is positive (negative). The Northern government is encouraged to subsidize its domestic firm's production (see Appendix E).

3.2. The Northern government's patent subsidy

The Northern government can subsidize its domestic firm's expenditures on patent instead of production. We denote φ such subsidy. The static profit flows expressions are the same as under free trade. Unlike production subsidy, it does not affect directly outputs, price and profit flows. But it reduces the patent filing cost. The real cost now equals: $(1-\varphi)k$. The Northern firm's discounted sum of profit flows is:

$$\max_{k\geq 0} \Pi(k) = \int_0^{T(k)} e^{-i\tau}\hat{\pi}\left(\underline{\mu}^*\right)\mathrm{d}\tau + \int_{T(k)}^{\infty} e^{-i\tau}\hat{\pi}(\overline{\mu}^*)\mathrm{d}\tau - (1-\varphi)k \quad (14)$$

The Northern firm selects k which maximizes its discounted sum of profit flows. The first order condition involves:

$$K(k) = \frac{\hat{\pi}\left(\underline{\mu}^*\right) - \hat{\pi}(\overline{\mu}^*)}{1-\varphi} \quad (15)$$

Proposition 2: The Northern government's patent subsidy increases the Northern firm's expenditures on patent k and the monopoly length T. It slows down the new technology diffusion. However, the subsidy which maximizes the Northern country's national welfare is negative. The Northern government is encouraged to tax its domestic firm's expenditures on patent. In that case, the effect on k becomes negative; it accelerates technology diffusion. Such results mean that patent involves social cost for the Northern country's welfare.

Proof: Thanks to (15), k decreases with $(1-\varphi)$ i.e. it increases with φ. The difference in profit does not depend on φ. Therefore, the subsidy increases the Northern firm's expenditures on patent and the monopoly length: $\mathrm{d}k/\mathrm{d}\sigma > 0$, $\mathrm{d}T/\mathrm{d}\varphi > 0$. It slows down the new technology diffusion.

Such subsidy reduces the Northern firm's total patent filing cost. It explains why the firm is encouraged to increase k. The monopoly length of the new technology increases when the total patent filing cost does not vary.

We can summarize the economic impact of φ (see Appendix C). Since it increases the monopoly length, it increases (decreases) the Northern (Southern) firm's discounted sum of profit flows. The subsidy does not affect directly price and outputs. But the Southern firm uses the old technology longer i.e. its marginal cost remains $c^*\left(\underline{\mu}^*\right)$ longer. Therefore, it decreases the Northern country's consumer surplus. It also reduces the Northern country's public revenues. The total effect on the Northern country's national welfare is negative. Then, the Northern government's optimal subsidy is negative: the Northern government taxes its domestic firm's expenditures on patent. In that case, technology diffusion accelerates as compared to free trade (See Appendix E).

Such results mean that patent leads social cost for the Northern country's national welfare. It completely contrasts with several previous studies which establish that there is optimal patent length. As we said in the general Introduction, the main reason is that we use a simple structure where the Northern firm's patent protects the new technology monopoly but does not encourage further innovations. The Northern firm is encouraged to discover the new technology before time 0 because of a competitive disadvantage with respect to the Southern firm. Furthermore, we consider patent filing cost in this paper for the Northern firm unlike previous studies. Then, the positive effect of patent on the discounted profit is lower.

3.3. The Southern government's production subsidy

The Southern government can implement policy instrument in order to accelerate technology diffusion. The only instrument is a production subsidy because the Southern firm does not implement patent or R&D investment. Consider that the Northern government remains under free trade. We denote s^* the Southern government's subsidy. The Northern firm's profit expression is the same as Equation (1). The Southern firm's profit expression becomes:

$$\pi^*(x, y, s^*) = yp(x+y) - c(\mu^*)y + s^*y - F^*, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (16)$$

Its reaction function becomes: $y(x) = [c^*(\mu^*) - p(x+y) - s^*]/p'$ with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. The effects of the subsidy s^* on outputs, price and profits are symmetrical to these of the subsidy s. The Southern government's subsidy increases (decreases) the Southern (Northern) country's producer surplus. It also increases the Northern country's consumer surplus by increasing the total supply and reducing the market price.

The Northern firm selects k which maximizes its discounted sum of profit flows. The first order condition gives the same expression as under free trade except that the profit flows now depend on the subsidy s^*.

Proposition 3: The Southern government's production subsidy generally accelerates technology diffusion by decreasing k and T as compared to free trade. Under linear decreasing demand function, the effect on k is certainly negative. But under nonlinear demand function, the effect is either positive or negative. However, k is generally decreased with s^* as compared to free trade. Furthermore, the subsidy which maximizes the Southern country's national welfare is positive. The Southern government is encouraged to subsidize its domestic firm's production.

Proof: We have to find the effect of the subsidy s^* on the difference in profit $\left[\hat{\pi}\left(\underline{\mu}^*, s^*\right) - \hat{\pi}\left(\overline{\mu}^*, s^*\right)\right]$. Using the Northern firm's maximum profit, the Envelope Theorem does not permit to find the effect of the Southern firm's technological endowment. We have: $\mathrm{d}\hat{\pi}(\mu^*, s^*)/\mathrm{d}s^* = -\lambda$. The effect of the Southern firm's technology on the parameter λ is unknown. Denoting $\hat{x}$ the Northern firm's level of output which permits to achieve the maximum profit $\hat{\pi}$, we have: $\hat{\pi}(\mu^*, s^*) = \pi[\hat{x}(\mu^*, s^*)]$ and $\hat{x}(\mu^*, s^*) = \operatorname{argmax} \pi(x, y)$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. Then, we get: $\mathrm{d}\hat{\pi}(\mu^*, s^*)/\mathrm{d}s^* = \{\partial\pi[\hat{x}(\mu^*, s^*)]/\partial\hat{x}(\mu^*, s^*)\}\{\mathrm{d}\hat{x}(\mu^*, s^*)/\mathrm{d}s^*\}$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. Thanks to (3), we have: $\pi[\hat{x}(\mu^*, s^*)] = -\hat{x}(\mu^*, s^*)^2 p' - F$. Hence, we find: $\partial\pi[\hat{x}(\mu^*, s^*)]/\partial\hat{x}(\mu^*, s^*) = -2\hat{x}(\mu^*, s^*)p' - \hat{x}(\mu^*, s^*)^2 p'' = -\hat{x}(\mu^*, s^*)\pi_{xx} > 0$. Moreover, the expression of $\mathrm{d}x/\mathrm{d}c^*$ leads: $\mathrm{d}\hat{x}(\mu^*, s^*)/\mathrm{d}s^* = \pi_{xy}/D < 0$. Then:

$$\frac{\mathrm{d}\hat{\pi}(\mu^*, s^*)}{\mathrm{d}s^*} = -\hat{x}(\mu^*, s^*)\frac{\pi_{xx}\pi_{xy}}{D} = -\frac{\hat{x}(\hat{x}p'' + 2p')(\hat{x}p'' + p')}{p'(\hat{X}p'' + 3p')} < 0, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (17)$$

Let us find the effect of the Southern firm's technological endowment on the previous expression. First consider linear inverse demand function: $p'' = 0$. The derivative p' is constant and does not depend on the southern firm's marginal cost. Then, the Southern firm's technological endowment does not influence $(\hat{x}p'' + 2p')(\hat{x}p'' + p')/\left[p'\left(\hat{X}p'' + 3p'\right)\right]$. Furthermore, the Northern firm's output increases with the Southern firm's marginal cost: $\hat{x}$ is stronger when the Southern firm uses the old technology. Therefore, the cost of s^* on the Northern firm's profit is stronger when $\mu^* = \underline{\mu}^*$. The subsidy s^* decreases the difference in profit and k.

Now consider nonlinear demand function. The problem is that now the Southern firm's technological endowment has an impact on $(xp'' + 2p')(xp'' + p')/[p'(Xp'' + 3p')]$. We cannot make a general conclusion on the effect of the Southern firm's technological endowment on $\mathrm{d}\hat{\pi}(s^*)/\mathrm{d}s^*$. However, the first order effect remains the effect on $\hat{x}(s^*)$. Hence, generally, the difference in profit decreases with the subsidy s^*. We are going to use two specific nonlinear demand functions:

- First consider a specific concave demand function: $p(X) = a - X^2$, with $p' = -2X < 0$ and $p'' = -2 < 0$. We can demonstrate easily that:
  $\frac{\mathrm{d}^2\hat{\pi}}{\mathrm{d}s^*\mathrm{d}c^*(\mu^*)} = -\frac{2a+c(\overline{\mu})-3c^*(\mu^*)+3s^*}{16[2a-c(\overline{\mu})-c^*(\mu^*)+s^*]^{3/2}} - \frac{3[10a-7c(\overline{\mu})-3c^*(\mu^*)+3s^*][2a-3c(\overline{\mu})+c^*(\mu^*)-s^*]}{64[2a-c(\overline{\mu})-c^*(\mu^*)+s^*]^{5/2}} < 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$
- Now consider a specific convex demand function: $p(X) = a - X^{1/2}$, with $p' = -(1/2)X^{-1/2} < 0$ and $p'' = (1/4)X^{-3/2} > 0$. We can demonstrate easily that:
  $\frac{\mathrm{d}^2\hat{\pi}}{\mathrm{d}s^*\mathrm{d}c^*(\mu^*)} = -\frac{32[a+2c(\overline{\mu})-3c^*(\mu^*)+3s^*]}{125} < 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$.

With these examples, the negative effect of the subsidy s^* on the Northern firm's profit is stronger when the Southern firm's technology is old. Hence, $\left[\pi\left(\underline{\mu}^*\right) - \pi(\overline{\mu}^*)\right]$ and k increases with s^*. Generally, the subsidy s^* accelerates technology diffusion. The reason is that the Southern government's subsidy involves a cost on the Northern firm's profit which is generally stronger when the Southern firm uses the old technology: the Northern firm decreases k to reduce such cost.

We can summarize the economic impact of s^* (see Appendix D). The Southern government's subsidy increases (decreases) the Southern (Northern) country's producer surplus by increasing (decreasing) output and profit. It reduces the market price and the total supply and increases the Northern country's consumer surplus. But it represents government expenditures: it reduces the Southern country's public revenues. The Southern country's national welfare increases with the subsidy until an optimal level. The Northern country's national welfare also increases with s^* because the positive effect on the consumer surplus is stronger than the negative effect on the Northern firm's discounted profit. The Southern government is encouraged to subsidize its domestic firm's production (see Appendix E).

4. Bertrand competition

Now, consider price competition at each point of time. Here, we use general forms again. In the Appendix, we use three specific demand functions (See Appendix F).

Assumption 2b: At each point of time, there is Bertrand competition. Firms produce slightly differentiated goods to avoid a "Bertrand paradox". We denote p (p^*) the Northern (Southern) firm's price. Each output depends on both prices: $x = x(p, p^*)$, $y = y(p, p^*)$. The domestic output decreases (increases) with the domestic (foreign) price: $x_p < 0$, $x_{p^*} > 0$, $y_p > 0$, $y_{p^*} < 0$. Own effects are still stronger than cross effects: $|x_p| > x_{p^*}$, $y_p < |y_{p^*}|$.

The static profit expressions are:

$$\pi(p, p^*) = px(p, p^*) - c(\overline{\mu})x(p, p^*) - F \quad (18)$$

$$\pi^*(p, p^*) = p^*y(p, p^*) - c^*(\mu^*)y(p, p^*) - F^*, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (19)$$

Each firm selects the price which maximizes its profit. The first order conditions give the following reaction functions:

$$p(p^*) = c(\overline{\mu}) - x(p, p^*)/x_p,\; p^*(p) = c^*(\mu^*) - y(p, p^*)/y_{p^*}, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (20)$$

Each profit increases with the foreign price: $\pi_{p^*} = x_{p^*}[p - c(\overline{\mu})] > 0$, $\pi_p^* = y_p.[p^* - c^*(\mu^*)] > 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$.

Assumption 3b: The second order conditions are verified: $\pi_{pp} = 2x_p + x_{pp}[p - c(\overline{\mu})] < 0$, $\pi^*_{p^*p^*} = 2y_{p^*} + y_{p^*p^*}[p^* - c^*(\mu^*)] < 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. However, cross effects are now positive: $\pi_{pp^*} = x_{pp^*}[p - c(\overline{\mu})] + x_{p^*} > 0$, $\pi^*_{p^*p} = y_{p^*p}[p^* - c^*(\mu^*)] + y_p > 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. Own effects are stronger than cross effects: $|\pi_{pp}| > \pi_{pp^*}$, $|\pi^*_{p^*p^*}| > \pi^*_{p^*p}$.

Stability condition is also verified under Bertrand competition: $B = \pi_{pp}\pi^*_{p^*p^*} - \pi_{pp^*}\pi^*_{p^*p} > 0$.

We have to find the effect of c^* on prices, outputs and profits as under Cournot competition. We can verify easily that each price increases with the Southern firm's marginal cost: $\mathrm{d}p/\mathrm{d}c^* = -y_{p^*}\pi_{pp^*}/B > 0$, $\mathrm{d}p^*/\mathrm{d}c^* = y_{p^*}\pi_{pp}/B > 0$. The effect on the Northern (Southern) firm's output is negative (uncertain): $\mathrm{d}x/\mathrm{d}c^* = y_{p^*}(x_{p^*}\pi_{pp} - x_p\pi_{pp^*})/B$, $\mathrm{d}y/\mathrm{d}c^* = y_{p^*}(y_{p^*}\pi_{pp} - y_p\pi_{pp^*})/B < 0$. The impact on the total supply is uncertain which means that the impact on the consumer surplus is also uncertain: $\mathrm{d}X/\mathrm{d}c^* = (x_p + y_p)\,\mathrm{d}p/\mathrm{d}c^* + (x_{p^*} + y_{p^*})\,\mathrm{d}p^*/\mathrm{d}c^*$. But considering symmetry of effects $x_p = y_{p^*}$ and $x_{p^*} = y_p$, then $\mathrm{d}X/\mathrm{d}c^* < 0$: the Southern firm's marginal cost decreases the Northern country's consumer surplus. Finally, let us search the effect on profits. Using the maximum profits, the Envelope Theorem gives:

$$\frac{\mathrm{d}\hat{\pi}(c^*)}{\mathrm{d}c^*} = \frac{\partial\pi(p, p^*)}{\partial c^*} - \gamma\frac{\partial\pi^*_{p^*}}{\partial c^*} = \theta y_{p^*} > 0 \quad (21)$$

$$\frac{\mathrm{d}\hat{\pi}^*(c^*)}{\mathrm{d}c^*} = \frac{\partial\pi^*(p,p^*,c^*)}{\partial c^*} - \gamma^*\frac{\partial\pi_p}{\partial c^*} = -\hat{y}(c^*) < 0 \tag{22}$$

The parameters γ and γ^* are Lagrange multipliers under Bertrand competition. Since $\partial\pi^*_{p^*}/\partial c^* = -y_{p^*} > 0$, γ and γ^* are necessarily negative: $\gamma < 0$, $\gamma^* < 0$. The Southern firm's marginal cost increases (decreases) the Northern (Southern) firm's profit.

The Northern firm selects k which maximizes its discounted sum of profit flows. The first order condition involves the same expression as Equation (10): k depends on the difference in profit again.

Let us study the effect of each production subsidy. The effect of the patent subsidy remains the same as under Cournot competition because it does not influence prices, outputs and profits directly (see Appendix H).

4.1. The Northern government's production subsidy

First consider a production subsidy s implemented by the Northern government. The Northern firm's first order condition gives: $p(p^*) = c(\overline{\mu}) - s - x(p,p^*)/x_p$. The Southern firm's reaction function remains the same as under free trade. As under Cournot competition, we find the effect of the subsidy on outputs, price and profits. Each price decreases with the subsidy. The effect on the Northern (Southern) firm's output is positive (uncertain). The effect on the total supply is uncertain, but considering symmetry of effects $x_p = y_{p^*}$ and $x_{p^*} = y_p$, then $\mathrm{d}X/\mathrm{d}s > 0$. In that case, the subsidy s increases the Northern country's consumer surplus. The Northern (Southern) firm's profit increases (decreases) with the subsidy. Note that the positive effect of the subsidy on the Northern firm's profit equals its equilibrium level of output as under Cournot Competition:

$$\frac{\mathrm{d}\hat{\pi}(\mu^*,s)}{\mathrm{d}s} = \frac{\partial\pi(p,p^*,s)}{\partial s} - \gamma\frac{\partial\pi^*_{p^*}}{\partial s} = \hat{x}(\mu^*,s) > 0, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \tag{23}$$

The Northern firm selects k which maximizes its discounted sum of profit flows. The first order condition remains the same.

Proposition 4: The Northern government's production subsidy increases the Northern firm's expenditures on patent k and the monopoly length T. It slows down the new technology diffusion.

Proof: The Northern firm's expenditures on patent k still increases with the difference in profit $[\hat{\pi}(\mu^*,s) - \hat{\pi}(\mu^*,s)]$. The subsidy increases the Northern firm's maximum profit; see Equation (23). The Northern firm's output increases with the Southern firm's marginal cost, hence: $\hat{x}\left(\underline{\mu}^*,s\right) > \hat{x}(\overline{\mu}^*,s)$. The subsidy s enhances the difference in profit. Then, it increases k and T. Therefore, the Northern government's production subsidy slows down technology diffusion. Bertrand competition does not modify the previous results. The economic impact of the Northern government's production subsidy is the same as under Cournot competition (see Appendix G). The Northern government is encouraged to subsidize its domestic firm's production (see Appendix J).

4.2. The Southern government's production subsidy

Consider now that the Southern government implements a production subsidy while the Northern government remains under free trade. The Southern firm's first order condition gives: $p^*(p) = c^*(\mu^*) - s^* - y(p,p^*)/y_{p^*}$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$. The Northern firm's reaction function remains the same as under free trade. According to the previous sub-section, we already know the impact of s^* on prices, outputs and profits. It reduces each price, has a positive (uncertain) effect on the Southern (Northern) firm's output and an uncertain effect on the total supply. But with symmetry of effects, it increases the total supply. In that case, the subsidy s^* increases the Northern country's consumer surplus. Finally, s^* increases (decreases) the Southern (Northern) firm's profit.

The Northern firm selects k which maximizes its discounted sum of profit flows. Let us search the effect of s^*.

Proposition 5: Generally, the Southern government's production subsidy decreases k and T, and accelerates the new technology diffusion. Under linear demand functions, it always reduces k. But under nonlinear demand functions, the effect can be either positive or negative. Nevertheless, the effect remains generally negative. However, the optimal subsidy which maximizes the Southern country's national welfare is negative. The Southern government is encouraged to tax its domestic firm's production. In that case, it slows down technology diffusion.

Proof: With the same demonstration as under Cournot competition, we have:

$$\frac{\mathrm{d}\hat{\pi}(\mu^*,s^*)}{\mathrm{d}s^*} = -[\hat{p}(\mu^*,s^*) - c(\overline{\mu})]\frac{\pi_{pp}\pi_{pp^*}y_{p^*}}{B} < 0, \text{ with } \mu^* = \underline{\mu}^*, \overline{\mu}^* \quad (24)$$

Let us find the effect of the Southern firm's technological endowment on the previous expression. First consider linear demand functions: $x_{pp} = 0$, $x_{pp^*} = 0$, $y_{p^*p} = 0$, $y_{p^*p^*} = 0$. Then, x_p, x_{p^*}, y_p and y_{p^*} are constant and $\pi_{pp}\pi_{pp^*}y_{p^*}/B$ does not depend on the Southern firm's technological endowment. Moreover, we already proved that $\mathrm{d}p/\mathrm{d}c^* > 0$. Therefore, the negative effect of the subsidy on the Northern firm's maximum profit is stronger when the Southern firm uses the old technology: the subsidy decreases k.

Now consider nonlinear demand functions: $\pi_{pp}\pi_{pp^*}y_{p^*}/B$ depends on the Southern firm's technological endowment. The total effect on $\mathrm{d}\hat{\pi}(\mu^*,s^*)/\mathrm{d}s^*$ is uncertain. But the first order effect remains the impact on $[\hat{p}(\mu^*,s^*) - c(\overline{\mu})]$ which is stronger when the Southern firm's technology is old. Let us use two examples:

- First consider two specific concave demand functions: $x(p,p^*) = (a-2p+p^*)^{1/2}$ and $y(p,p^*) = (a+p-2p^*)^{1/2}$, with $x_p = -(a-2p+p^*)^{-1/2} < 0$, $x_{pp} = -(a-2p+p^*)^{-3/2} < 0$, $x_{p^*} = (1/2)(a-2p+p^*)^{-1/2} > 0$ and $x_{p^*p^*} = -(1/4)(a-2p+p^*)^{-3/2} > 0$. We can demonstrate easily that:

  $\frac{\mathrm{d}^2\hat{\pi}}{\mathrm{d}s^*\mathrm{d}c^*(\mu^*)} = -\frac{3}{64\{2[4a-5c(\overline{\mu})+c^*(\mu^*)-s^*]\}^{1/2}} < 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$

- Now consider two specific convex demand functions: $x(p,p^*) = (a-2p+p^*)^2$ and $y(p,p^*) = (a+p-2p^*)^2$, with $x_p = -4(a-2p+p^*) < 0$, $x_{pp} = 8 > 0$, $x_{p^*} = 2(a-2p+p^*) > 0$ and $x_{p^*p^*} = 2 > 0$. We can demonstrate easily that:

  $\frac{\mathrm{d}^2\hat{\pi}}{\mathrm{d}s^*\mathrm{d}c^*(\mu^*)} = -\frac{1536[7a-11c(\overline{\mu})+4c^*(\mu^*)-4s^*]}{42875} < 0$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$

With these examples, the negative effect of s^* on the Northern firm's equilibrium profit is stronger when the Southern firm's technology is old. Generally, the Southern government subsidy s^* reduces k and the monopoly length T. It accelerates technology diffusion. Then, the results hold both under Cournot and Bertrand competition.

But now, the Southern government is encouraged to tax its domestic firm's production (see Appendix I and Appendix J). The positive effect of the subsidy on the Southern firm's profit is lower than the Southern government's expenditures. The main reason is that under Bertrand competition, the Southern firm's profit increases with the Northern firm's price while the subsidy s^* decreases such price. Under Cournot competition, the Southern firm's profit decreases with the Northern firm's output while the subsidy s^* decreases such output. It explains why the Southern firm's profit gain is lower under Bertrand competition.

5. Discussion

Table 1 summarizes the results presented in this paper. The following results hold under twelve cases by using specific linear, concave and convex demand functions, specific linear and concave monopoly length functions and several values for the production marginal costs. See Appendix.

Table 1 – Economic impact of each public policy instrument

Cournot competition								
Instrument	T	Π	Π^*	CS	PR	PR^*	W	W^*
s	+	+	−	+	−	0	+	−
φ	+	+	−	−	−	0	−	−
s^*	−	−	+	+	0	−	+/−	+
Bertrand competition								
Instrument	T	Π	Π^*	CS	PR	PR^*	W	W^*
s	+	+	−	+	−	0	+	−
φ	+	+	−	−	−	0	−	−

s^*	$-$	$-$	$+$	$+$	0	$-$	$+/-$	$-$

Source: author

Note: CS denotes the Northern country's consumer surplus, PR (PR^*) the Northern (Southern) government's public revenues and W (W^*) the Northern (Southern) country's national welfare.

The Northern government's production subsidy slows down technology diffusion by increasing the monopoly length of the new technology. It also increases (decreases) the Northern (Southern) firm's discounted sum of profit flows. The consumer surplus increases with the subsidy but public revenues decreases. The effect on the Northern (Southern) country's national welfare is positive (negative).

The Northern government's patent subsidy also slows down technology diffusion. Unlike the production subsidy, it reduces the Northern country's consumer surplus and national welfare as compared to free trade. The Northern government is encouraged to tax its domestic firm's expenditures on patent. Note that production subsidy is always better than patent tax. Production subsidy increases consumer surplus, monopoly length and the Northern firm's profit while the patent tax only increases consumer surplus by reducing monopoly length (See Appendix E and J). Then, the Northern government's favorite policy instrument is the production subsidy.

Therefore, patent involves social cost for the Northern country's national welfare. It contrasts with previous studies which prove that there is optimal patent length. As the matter of fact, here we use a simple structure where the Northern firm's patent represents technology protection but does not encourage further inventions or innovations. The Northern firm is encouraged to discover the new technology before time 0 because of a competitive disadvantage with respect to the Southern firm. Furthermore, we consider patent filing cost in this paper for the Northern firm unlike previous studies which analyze optimal patent length. Then, the positive effect of patent on the discounted profit is lower.

The Southern government's production subsidy is a way to accelerate technology diffusion when the Northern country remains under free trade. Generally, the impact on the monopoly length of the new technology is negative. It increases the Southern country's welfare under Cournot competition. However, under Bertrand competition, the effect is negative. In that case, the Southern government does not implement such instrument. The optimal instrument becomes tax. It may increase the Northern country's national welfare by increasing the consumer surplus. As a matter of fact, the favorite instrument of the Northern country's consumer is the Southern government's production subsidy. Under Bertrand competition, it increases the Northern country's national welfare for eleven cases over twelve. Under Cournot competition, it either increases or decreases the Northern welfare.

6. Concluding remarks

The objective of this paper is to study the impact of public policy instruments on technology diffusion in a dynamic North-South model. Here we have explored a case where the Northern firm implements patent in order to increase the monopoly length of the new technology. Then, we demonstrate that developing countries can accelerate technology diffusion by implementing public policy. However, developed countries can slow it down in the same way.

Sometimes, developing countries cannot subsidize their industries because government expenditures would be too high with respect to government revenues as this paper demonstrates. Now, if developed countries aim to help developing countries by accelerating technology diffusion, liberalization is one way. Then, the role of the WTO across TRIPs agreement and trade liberalization is crucial to promote access to technological information for developing countries.

Note that we omit externalities of technology diffusion in developing countries in the welfare analysis in order to study a simple case. Sometimes, negative externalities appear. For example, there is a cost for non-qualified labor because modern technologies involve that firms of developing countries need to hire qualified labor. Another example is environment. Technology diffusion may create pollution in developing countries.

Last, let us mention a few directions of possible extensions of this model. It would be interesting to analyze the relationship between public policies and technology by considering innovation in terms of product instead of process. We could also add a market in the Southern country. In that case, both firms would sell a part of their output on the Southern market. Finally, we could consider other form of technology protection like trade secret which also permits to increase the monopoly length of the new technology.

Appendix

A. General Framework under Cournot competition

First, consider the following linear demand function: $p(X) = a - X$. The Northern firm selects k which maximizes its discounted sum of profit flows. Consider an iso-elastic function of monopoly length: $T(k) = \theta + k^{\omega}$, with $0 < \omega < 1$ and θ the part of T which does not depends on k. We use numerical values. Set: $a = 100$, $c(\overline{\mu}) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $c^*\left(\overline{\mu}^*\right) = 3$, $i = 0.1$, $\theta = 5$ and $\omega = 1$. Then, the equilibrium value of expenditures on patent is: $\hat{k} = 36.846$. The monopoly length of the new technology equals: $\hat{T} = 41.846$. Setting $F = 10$ and $F^* = 10$, the discounted sums of profit flows are: $\widehat{\Pi} = 10{,}964.265$ and $\widehat{\Pi}^* = 9{,}120.197$.[5] The Northern country's total consumer surplus equals: $CS = 20{,}276.994$.[6] Therefore, under free trade, the national welfares are: $W = 31{,}241.259$ and $W^* = 9{,}120.197$.[7]

Now, consider the following concave function: $p(X) = a - X^2$. Using the same numerical values, we find: $\hat{k} = 13{,}91$. The monopoly length of the new technology equals: $\hat{T} = 18.91$. The discounted profits are: $\widehat{\Pi} = 1{,}631.166$ and $\widehat{\Pi}^* = 1{,}469.059$. The Northern country's consumer surplus equals: $CS = 2{,}507.59$. The national welfares are: $W = 3{,}838.745$, $W^* = 1{,}469.059$.

Finally, consider the following convex function: $p(X) = a - X^{1/2}$. Using the same numerical values, we have: $\hat{k} = 82.689$. The monopoly length of the new technology equals: $\hat{T} = 87.689$. The discounted profits are: $\widehat{\Pi} = 648{,}229.391$ and $\widehat{\Pi}^* = 473{,}230.508$. The Northern country's consumer surplus equals: $CS = 1{,}486{,}490.21$. The national welfares are: $W = 2{,}134{,}719.601$, $W^* = 473{,}230.508$.

We also consider a case where $a = 20$ instead of 100 and another where $\omega = 0.5$ instead of 1. The following results hold under these cases.

B. The Northern government's production subsidy under Cournot Competition

Table 2 – Economic impact of the Northern government's production subsidy such as $s = 5$ as compared to free trade under Cournot competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔPR
Linear demand function	$a = 100$	$\omega = 0.5$	2,316.449	-1,014.15	1,071.802	-1,820.95
		$\omega = 1$	2,332.366	-986.014	1,074.33	-1,832.642
	$a = 20$	$\omega = 0.5$	526.697	-130.56	187.882	-479.469
		$\omega = 1$	551.234	-103.264	183.906	-497.368
Concave demand function	$a = 100$	$\omega = 0.5$	248.868	-104.56	86.999	-187.214
		$\omega = 1$	250.643	-103.485	86.495	-188.711
	$a = 20$	$\omega = 0.5$	121.607	-35.451	36.684	-102.773
		$\omega = 1$	124.699	-36.276	34.535	-105.855
Convex demand function	$a = 100$	$\omega = 0.5$	224,774.361	-91,986.581	119,766.372	-184,951.36
		$\omega = 1$	224,889.728	-91,728.553	119,820.855	-185,023.1
	$a = 20$ (a)	$\omega = 0.5$	6,811.031	-470.089	2,269.309	-5,156.316
		$\omega = 1$	5,889.442	-598.863	2,027.709	-5,220.472

Source: author's calculations

[5] The Southern firm's discounted sum of profit flows is: $\Pi^*(k) = \left\{\left[1 - e^{-i(\theta+k^\omega)}\right]\hat{\pi}\left(\underline{\mu}^*\right) + e^{-i(\theta+k^\omega)}\hat{\pi}\left(\overline{\mu}^*\right)\right\}/i$.

[6] At each point of time, the consumer surplus is: $\int_0^{\hat{X}(\mu^*)} p[X(\mu^*)]\mathrm{d}X(\mu^*) - \hat{X}(\mu^*)p\left[\hat{X}(\mu^*)\right]$, with $\mu^* = \underline{\mu}^*, \overline{\mu}^*$.

[7] $W = \Pi + CS + PR$ and $W^* = \Pi^* + PR^*$ where PR and PR^* denote public revenues (which equals zero under free trade). Note that the Northern firm transfers its profit to its headquarters in the Northern country. This is the reason why the Northern country's national welfare increases with such profit.

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$. (a) Note that under convex demand function, when $a = 20$, with $s = 5$, the Southern firm leaves the market. Then, the results correspond to the economic impact of a production subsidy such as $s = 3$.

C. The Northern government's patent subsidy under Cournot competition

Table 3 – Economic impact of the Northern government's patent subsidy such as $\varphi = 0.5$ as compared to initial situation without patent subsidy under Cournot competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔPR
Linear demand function	$a = 100$	$\omega = 0.5$	50,522	-95.706	-48.998	-71,979
		$\omega = 1$	19.957	-9.543	-4.886	-21.889
	$a = 20$	$\omega = 0.5$	6.578	-14.509	-8.421	-10.966
		$\omega = 1$	11.596	-7.568	-4.392	-13.527
Concave demand function	$a = 100$	$\omega = 0.5$	2.523	-8.716	-3.215	-4.447
		$\omega = 1$	8.489	-10.644	-3.926	-10.421
	$a = 20$	$\omega = 0.5$	0.642	-2.073	-0.824	-1.196
		$\omega = 1$	4.488	-9.028	-3.591	-6.42
Convex demand function	$a = 100$	$\omega = 0.5$	864.958	-577.198	-370.652	-998.655
		$\omega = 1$	42.879	-8.614	-5.532	-44.81
	$a = 20$	$\omega = 0.5$	138.106	-123.42	-99.812	-181.032
		$\omega = 1$	25.778	-5.965	-4.824	-6.756

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$.

D. The Southern government's production subsidy under Cournot Competition

Table 4 – Economic impact of the Southern government's production subsidy such as $s^* = 5$ as compared to free trade under Cournot competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔW	ΔPR^*
Linear demand function	$a = 100$	$\omega = 0.5$	-1,074.595	2,176.516	1,083.419	8.825	-1,709.864
		$\omega = 1$	-1,082.812	2,136.493	1,075.79	-7.022	-1,684.938
	$a = 20$	$\omega = 0.5$	-178.716	426.049	197.446	18.73	-398.714
		$\omega = 1$	-191.294	375.976	191.216	-0.077	-361.111
Concave demand function	$a = 100$	$\omega = 0.5$	-107.735	242.845	87.472	-20.263	-181.986
		$\omega = 1$	-108.872	240.363	87.44	-21.432	-179.621
	$a = 20$	$\omega = 0.5$	-42.913	108.394	37.298	-5.615	-91.722
		$\omega = 1$	-44.209	111.175	39.191	-5.018	21.288
Convex demand function	$a = 100$	$\omega = 0.5$	-105,908.9	192,439.5	119,913.1	14,004.12	30,720.96
		$\omega = 1$	-105,956.2	192,088.1	119,823.2	13,867.01	-161,506.8
	$a = 20$	$\omega = 0.5$	-3,179.313	6,319.244	3,803.59	624.277	-7,232.379
		$\omega = 1$	-3,296.257	5,497.334	3,608.448	312.192	-6,649.162

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$.

E. Welfare analysis under Cournot competition

Table 5 – Values of the national welfares when governments implement optimal instrument under Cournot competition

	Instrument	Optimal	W	W^*

				value of instrument		
Linear demand function	$a = 100$	$\omega = 0.5$	s	90 (a)	46,681.423	-90.873
			φ	-43	31,269.433	9,829.053
			s^*	24	32,011.317	10,661.876
		$\omega = 1$	s	90 (a)	46,831.013	-98.602
			φ	-38	31,269.214	9,845.477
			s^*	23	31,900.085	10,279.058
	$a = 20$	$\omega = 0.5$	s	10 (a)	1,249.563	-78.895
			φ	-7	869.433	142.713
			s^*	4	871.62	150.342
		$\omega = 1$	s	10 (a)	1,250.97	-96.811
			φ	-6	869.156	140.39
			s^*	3	846.431	71.111
Concave demand function	$a = 100$	$\omega = 0.5$	s	184 (a)	6,246.854	-98.709
			φ	-4	3,846.35	1,530.075
			s^*	49	3,890.387	1,808.939
		$\omega = 1$	s	184 (a)	6,239.984	-99.821
			φ	-3	3,846.165	1,532.92
			s^*	48	3,871.575	1,761.204
	$a = 20$	$\omega = 0.5$	s	24 (a)	353.329	-95.871
			φ	-3	192.482	2.047
			s^*	8	189.641	19.014
		$\omega = 1$	s	24 (a)	343.101	-98.89
			φ	-0.8	192.405	2.66
			s^*	10	194.17	18.5249
Convex demand function	$a = 100$	$\omega = 0.5$	s	43 (a)	3,026,499.65	251.979
			φ	(b)	2,138,948.73	540,415.32
			s^*	11	2,181,284.76	517,811.01
		$\omega = 1$	s	43 (a)	3,028,554.62	200.086
			φ	(b)	2,138,948.73	540,415.32
			s^*	11	2,182,306.7	516,137.43
	$a = 20$	$\omega = 0.5$	s	3 (a)	13,897.634	95.302
			φ	-27	11,462.408	1,795.259
			s^*	1	11,222.785	962.508
		$\omega = 1$	s	3 (a)	14,150.733	-52.052
			φ	-27	11,478.088	868.938
			s^*	1	11,359.188	589.289

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$. (a) The value of $\hat{s}$ corresponds to the maximum subsidy. With a stronger subsidy, the Southern firm's output becomes negative. (b) In that case, we find an optimal patent tax such as expenditures on patent k equals zero.

F. General framework under Bertrand competition

First, consider the following linear demand functions: $x(p, p^*) = a - 2p + p^*$, $y(p, p^*) = a + p - 2p^*$. The Northern firm selects k which maximizes its discounted sum of profit flows. Set the same numerical values as under Cournot competition. We find: $\hat{k} = 34.523$. The monopoly length of the new technology is: $\hat{T} = 39.523$. The discounted sums of profit flows are: $\hat{\Pi} = 21{,}284.9$ and $\hat{\Pi}^* = 19{,}071.454$. The Northern country's consumer surplus equals: $CS = 20{,}295.438$. The national welfares are: $W = 41{,}580.338$ and $W^* = 19{,}071.454$.

Now, consider the specific concave demand functions: $x(p,p^*) = (a - 2p + p^*)^{1/2}$, $y(p,p^*) = (a + p - 2p^*)^{1/2}$. Using the same numerical values, we have: $\hat{k} = 8.673$. The monopoly length of the new technology equals: $\hat{T} = 13.673$. The discounted profits are: $\hat{\Pi} = 3{,}298.21$ and $\hat{\Pi}^* = 3{,}133.09$. The Northern country's consumer surplus equals: $CS = 2{,}213.324$. The national welfares are: $W = 5{,}511.534$, $W^* = 3{,}133.09$.

Finally, consider the specific convex demand functions: $x(p,p^*) = (a - 2p + p^*)^2$, $y(p,p^*) = (a + p - 2p^*)^2$. Using the same numerical values, we find: $\hat{k} = 82.489$. The monopoly length of the new technology equals: $\hat{T} = 87.489$. The discounted profits are: $\hat{\Pi} = 1{,}231{,}068.137$ and $\hat{\Pi}^* = 1{,}005{,}961.454$. The Northern country's consumer surplus equals: $CS = 1{,}491{,}541.387$. The national welfares are: $W = 2{,}722{,}609.524$, $W^* = 1{,}005{,}961.454$.

G. The Northern government's production subsidy under Bertrand competition

Table 6 – Economic impact of the Northern government's production subsidy such as $s = 5$ as compared to free trade under Bertrand competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔPR
Linear demand function	$a = 100$	$\omega = 0.5$	3,153.438	-841.426	1,158.291	-3,495.517
		$\omega = 1$	3,163.307	-818.947	1,172.527	-3,505.95
	$a = 20$	$\omega = 0.5$	657.685	-130.705	265.296	-821.954
		$\omega = 1$	671.763	-116.166	279.473	-836.951
Concave demand function	$a = 100$	$\omega = 0.5$	332.215	-64.816	89.155	-359.847
		$\omega = 1$	332.579	-65.629	89.087	-360.233
	$a = 20$	$\omega = 0.5$	149.712	-25.765	41.342	-171.64
		$\omega = 1$	149.761	-31.884	39.804	-171.794
Convex demand function	$a = 100$	$\omega = 0.5$	317,905.809	-90,914.53	151,392.54	-363,312.36
		$\omega = 1$	318,026.453	-90,581.823	151,630.783	-363,437.44
	$a = 20$	$\omega = 0.5$	15,377.303	-1,622.529	9,273.359	-22,343.803
		$\omega = 1$	15,616.79	-1,313.533	9,540.364	-22,586.827

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$.

H. The Northern government's patent subsidy under Bertrand competition

Table 7 – Economic impact of the Northern government's patent subsidy such as $\varphi = 0.5$ as compared to initial situation without patent subsidy under Bertrand competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔPR
Linear demand function	$a = 100$	$\omega = 0.5$	39.849	-142.533	-50.325	-57.913
		$\omega = 1$	18.796	-17.016	-6.008	-20.727
	$a = 20$	$\omega = 0.5$	4.373	-19.862	-6.58	-7.478

		$\omega = 1$	10.231	-14.815	-4.908	-12.162
Concave demand function	$a = 100$	$\omega = 0.5$	1.046	-8.981	-2.388	-1.916
		$\omega = 1$	5.871	-24.709	-6.57	-7.802
	$a = 20$	$\omega = 0.5$	0.22	-1.916	-0.502	-0.421
		$\omega = 1$	1.562	-23.311	-6.104	-3.494
Convex demand function	$a = 100$	$\omega = 0.5$	858.28	-857.971	-349.66	-991.374
		$\omega = 1$	42.779	-12.864	-5.243	-44.71
	$a = 20$	$\omega = 0.5$	137.977	-192.666	-59.523	-180.876
		$\omega = 1$	25.772	-9.318	-2.879	-27.703

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$.

I. The Southern government's production subsidy under Bertrand competition

Table 8 – Economic impact of the Southern government's production subsidy such as $s^* = 5$ as compared to free trade under Bertrand competition

			$\Delta\Pi$	$\Delta\Pi^*$	ΔCS	ΔW	ΔPR^*
Linear demand function	$a = 100$	$\omega = 0.5$	-860.923	3,038.373	1,088.069	227.146	-3,367.233
		$\omega = 1$	-863.793	2,999.275	1,067.637	203.844	-3,329.412
	$a = 20$	$\omega = 0.5$	-147.677	574.557	212.977	65.3	-729.144
		$\omega = 1$	-151.683	526.615	186.863	35.18	-676.827
Concave demand function	$a = 100$	$\omega = 0.5$	-65.193	328.865	87.886	22.692	-356.259
		$\omega = 1$	-65.267	327.307	87.325	22.059	-354.357
	$a = 20$	$\omega = 0.5$	-27.08	142.465	38.457	11.377	-164.437
		$\omega = 1$	-27.063	143.051	38.644	11.581	-164.83
Convex demand function	$a = 100$	$\omega = 0.5$	-103,800.8	279,917.55	117,386.87	13,586.084	-321,315.4
		$\omega = 1$	-103,846.1	279,464.12	117,078.31	13,232.237	-320,920.1
	$a = 20$	$\omega = 0.5$	-3,190.48	9,111.974	3,911.005	720.524	-14,143.5
		$\omega = 1$	-3,282.198	8,130.777	3,230.064	-52.134	-13,029.29

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$.

J. Welfare analysis under Bertrand competition

Table 9 – Values of the national welfares when governments implement optimal instrument under Bertrand competition

			Instrument	Optimal value of instrument	W	W^*
Linear demand function	$a = 100$	$\omega = 0.5$	s	34	44,562.881	14,203.016
			φ	(a)	41,676.49	20,111.871
			s^*	-4	41,503.288	19,630.141
		$\omega = 1$	s	34	44,640.216	13,858.797
			φ	-30	41,675.154	20,092.407
			s^*	-4	41,514.913	19,156.38
	$a = 20$	$\omega = 0.5$	s	6	1,285.875	244.28
			φ	-53	1,185.129	431.903
			s^*	-1	1,183.596	399.59
		$\omega = 1$	s	7	1,292.519	137.678

			φ	-4	1,183.956	412.242
			s^*	-1	1,178.107	293.967
Concave demand function	$a = 100$	$\omega = 0.5$	s	55	5,844.52	2,501.141
			φ	(a)	5,524.542	3,201.316
			s^*	-8	5,494.909	3,207.839
		$\omega = 1$	s	55	5,834.095	2,441.136
			φ	-1.3	5,523.186	3,197.333
			s^*	-8	5,484.875	3,150.853
	$a = 20$	$\omega = 0.5$	s	10	336.201	64.861
			φ	(a)	311.586	117.403
			s^*	-1	310.15	116.308
		$\omega = 1$	s	10	333.886	57.088
			φ	-0.005	311.578	17.376
			s^*	-1	310.306	117.931
Convex demand function	$a = 100$	$\omega = 0.5$	s	20	2,983,220.93	677,541.219
			φ	(a)	2,724,546.6	1,104,306.33
			s^*	-2	2,726,969.48	1,010,299.45
		$\omega = 1$	s	20	2,985,353.98	676,563.042
			φ	(a)	2,724,546.6	1,104,306.33
			s^*	-2	2,728,494.83	1,008,352.29
	$a = 20$	$\omega = 0.5$	s	4	17,515.205	1,448.673
			φ	-1	15,164.473	3,079.339
			s^*	-0.2	15,241.296	2,823.633
		$\omega = 1$	s	4	18,025.877	1,187.289
			φ		15,414.681	2,310.893
			s^*	-0.2	15,550.562	2,297.96

Source: author's calculations

Note: $c(\overline{\mu}) = c^*(\overline{\mu}^*) = 3$, $c^*\left(\underline{\mu}^*\right) = 6$, $F = F^* = 10$, $i = 0.1$ and $\theta = 5$. (a) In that case, we find an optimal patent tax such as expenditures on patent k equals zero.

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