A proof that diversification generally pays, even with contagion

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Abstract

Portfolio diversification makes investors individually safer but creates connections between them through common asset holdings. The recent crisis has showed that such connections may enhance systemic risk by propagating shocks swiftly through the system. From this perspective, is diversification still desirable? We provide a simple model in which shocks spread though constrained selling from N diversified portfolio investors. We then compare the implied losses for every level of diversification. When agents follow a linear rule there may be a region on the parameter set for which the propagation effect dominates the individually safer one, but allowing for panic this region becomes trivial. We conclude that systemic risk should decrease with diversification.

This paper is in its preliminary stages, comments are very welcome.

1 Introduction

When thinking about financial markets and systemic risk, one can find it useful to consider a group of climbers roped together at the top of a cliff. Each climber individually favors being roped as it lowers his chances of falling off, yet one climber tripping now threatens the stability of his neighbors. The effect of being roped on the probability that many or all climbers fall is thus a priori ambiguous.

Prior to the 2008 credit crunch, both market participants and regulatory instances seemed to favor the roped equilibrium, implicitly assuming that individual soundness leads to systemic soundness. Yet the swiftness with which risk spread throughout the market, largely unanticipated, led to a shift to a more “connection-based” approach. Basel III will apply extra capital ratio requirement of up to 2.5% to well connected establishments. Measures of systemic importance that account for externalities, such as Covar (Adrian and Brunnermeier, 2011), or Shapley values (Tarashev et al., 2010) have recently gained in popularity.

This change in focus has implications on the desirability portfolio diversification from society’s perspective. Indeed while the individually risk reducing effect of diversification has been known since Markowitz (1952), diversification also forms “connections” between investors through common asset holdings, identified as a major carrier of contagion in the presence of fire sales (Shleifer and Vishny, 2012). The goal of this paper is to take a first step towards quantifying this contagion externality and comparing it to its individual risk-reducing effect, in order to get a primary assessment of diversification’s total impact on systemic risk, as well as the factors on which this impact may depend.

Systemic risk is studied by looking at the multivariate distribution of investors wealth, from which we draw the probability that a large number of investors fall below a given bankruptcy threshold K. This approach allows us to map the individually risk reducing effect and the contagion externality into two distinct components of systemic risk. An individually safer investor implies a lower probability that one investor goes bankrupt, while a higher contagion externality means a higher likelihood that a large number X of investors fall, conditional on one bankruptcy. Therefore the former affects the marginal distribution of each element on the vector of investors wealth, while the latter is contained within the dependence structure between all N investors.

We find intermediate numbers of bankruptcy are less likely with a higher degree of diversification, but the probability that investors fail simultaneously is larger. In a context of high selling constraint and weak demand, this probability of the "extreme failure" outcome becomes non-trivial, so that no or little diversification may become optimal for society. Going further, we introduce heuristics in the model by allowing agents on the demand side to grow more risk averse in a high variance environment. We find this strongly enhances the desirability of higher levels of diversification, as spreading shocks across assets lowers the scope for panic. Ultimately, the paper argues that over a reasonable set of parameters a high level of diversification should increase the resilience of the financial markets.

Previous work on the link between diversification and systemic is split between between two strands of literature which have rarely crossed[1].

The first concerns small-scale[2] models of financial contagion, which highlight different channels through which common asset holding may lead to fire sales, which may in turn degenerate into systemic events. Schinasi and Smith (2000) for instance show that routine portfolio rebalancing brings contagion. The scope is increased when agents are subject to wealth effects as in Kyle and Xiong (2001). Pauzner and Goldstein (2004) point out that fire sales may also result from strategic risk. Others authors relate diversification to different amplification mechanisms, which deserve to be mentioned although they will not feature as such in this paper. Market distress may lead to an even wider collapse if it turns into liquidity distress: Shin (2008) shows that leverage is negatively correlated to the market value of his assets. Allen et al. (2010) find that when investors need to roll debt over, being connected brings a negative reputation externality. Calvo and Mendoza (2000) argue that diversification lowers the incentive for investors to acquire information about securities before selling.

The second strand is based on a statistical approach. One method, taken by Shaffer (1994) or Wagner (2010),, is to compare statistically a fully diversified situation to a fully undiversified one. Both authors show that the risk that all investors fail simultaneously is necessarily higher in the fully diversified situation. Our study confirms this fact but also goes more in depth by considering any level of diversification and any number of failures. A second approach deals with "fat tails", which may mitigate the strength of the variance reducing effect, as showed Samuelson (1967). In particular, Ibrahimov et al. (2011) use an indicator of the tail behavior of returns to define a diversification threshold. They find that on a given parameter range there can exist a wedge between investors interests and society ones.

The crucial link between both facets of literature is the correlation structure between asset returns. In any contagion model the actions of investors endogenously create correlations between asset returns, as even two securities which are "fundamentally" independent become linked through the investors who hold them. Thus asset correlations are the output of the "contagion model" strand, and the input of the "statistical approach" one. The

[1]To a lesser extend our analysis may be linked to network analysis of financial stability. Our results are consistent with a "robust yet fragile" financial system, in line with Nier et al. (2007) or Amini et al (2012). Caccioli et al. (2012) use network analysis to find that overlapping portfolios may enhance systemic risk when leverage is high.

[2]The models usually feature only 2 assets, and diversification is defined as how evenly an investor spreads his wealth across both. A notable exception is provided by Lagunoff and Shreft (1999) who try moving to a 3 assets case. They find that the scope for contagion is decreased.

broad method of this paper is to bridge these two approaches, by proceeding in two steps:

1) setting up a larger scale contagion model. N constrained portfolio investors who hold from 1 to N assets interact with "convergence traders". This impacts prices, further tightening the constraint, and so on. Mathematically this translates into a linear system of N recurrence equations of price returns, in which the strength of the recurrence will depend on the magnitude of the constraint and the discount on the sales. Using the properties of circulant matrices, we are able to solve analytically the system, obtaining well-behaved N by N covariance matrices between assets and investors. Circulant matrices are a powerful tool, a side goal of the paper is to contribute to widening their use and understanding beyond the fields of network analysis and signal theory in which they are more common.

2) using the correlations between assets and investors to run a statistical analysis. The multivariate distribution of portfolio losses gives us the likelihood that any number of investors between 0 and N fail, for a given level of diversification n. To obtain more clear-cut conclusions, we attach to each outcome a cost that grows exponentially with the number of failures. This allows us to define a diversification threshold above which the benefits of individual soundness outweigh the costs of increased connectivity.

To the best of our knowledge two recent papers have used a similar approach. Danielsson, Shin, and Zigrand, define in 2011 a multivariate model which produces a matrix of covariance of unrestricted dimensions. The correlations obtained have a "fundamental" and an "endogenous risk" component, where endogenous risk is the risk resulting from "the actions of market participants which are hard wired in the system". Cont and Wagalath (2012) specify a similar but more aggregated model, which they calibrate to estimate the realized covariance matrix during well-know fire sales episodes, such as the aftermath of the collapse of Lehman brothers.

With respect to methodology, our study differs from Cont et al. through is micro foundations, and from Danielsson et al. in that their framework implies similar correlations across all assets, while in our case correlations between two assets i and j will depend on the "distance" i-j in the financial network. In spirit, both papers focus on explaining the pattern of prices during crisis episodes, while our interest lies with the desirability of diversification from a systemic perspective.

Section 2 presents the baseline model, and how it brings recurrence in asset prices. Section 3 solves this system and defines the realized covariances between assets and investors. Section 4 presents the distributions of investor wealth for the baseline model, and that with possible panics. Section 5 provides three extensions: a description of the patterns of contagion in the model, a study of the impact of a change in the financial network, which may be seen as a robustness test. Section 6 concludes.

2 Set-up

2.1 The market

The investment period starts at t=*0* and finishes at t=T, where each period $t \mapsto t+1$ may be seen as "a day" on the markets, and T is a large but finite number. Financial markets are composed of N risky assets which *fundamentally* follow an arithmetic Brownian motion, where the drift μ_i represents the fundamental evolution of asset i, and the normally distributed random variable ε_i^F is the shock stemming from fundamental factors. Across assets and time, these moments are equal and independent so that $\forall i,\ \forall t:\ \mu_{i,t} = \mu$ and $\Sigma_F = diag(\sigma_F^2)$. These assumptions will simplify presentation but also show how contagion may still occur through endogenous risk only, even though assets are symmetric and fundamentally independent.

The interaction between market participants will add another source of variation ε^* to prices. The dynamics of ε^* will depend on the behavior of investors which we derive later. For now we may just say that some investors face a constraint which allows them to buy/force them to sell in response to a positive/ negative shock on their wealth. The vector of shocks from market trading ε^*_{t+1} will thus depend on shocks at the previous period, both fundamental or from constrained trading $\varepsilon^*_{t+1} = f(\varepsilon^*_t, \varepsilon^F_t)$, where as we shall see f is a linear function.

Importantly, market trading shocks follow an error-correction process: $MR_{i,t+1} = -\lambda(p_{i,t} - p^F_{i,t})$, where $p^F_{i,t} = p_{i,0} + t\mu + \sum_{j=0}^{j=t} \varepsilon^F_{i,j}$ is the fundamental value of asset i at time t. We set a fairly low value for the speed of mean reversion λ, so that the deviation from fundamental value stemming from market trading at t is expected to have fully returned to 0 in the medium-run.

The intuition is that markets are subject to cycles of over/under investment, but eventually returns to a long-run equilibrium level. When investment is above its equilibrium value, leverage and risk are high, which leads investors to either be wiped out or get rid of their risk overhang in the following periods. When leverage is too low, asset prices are undervalued, which encourages investment. The MR component of price is thus a function of the underlying mean reverting investment, which we set to be exogenous :$MR_{i,t+1} = f(\triangle q^{MR}_{i,t})$.

Mathematically, the vector of asset prices writes:

$$\mathbf{P_{t+1}} = \mu + \mathbf{P_t} + \varepsilon^F_{t+1} + MR_{t+1} + \varepsilon^*_{t+1} \tag{1}$$

$$\Longrightarrow \triangle \mathbf{P_t} = \mu + \varepsilon^F_{t+1} - \lambda(\mathbf{P^F_t} - \mathbf{P_t}) + f(\varepsilon^*_t, \varepsilon^F_t) \tag{2}$$

where $\mathbf{P}$ and $\mathbf{Q}$, in bold, represent vectors. When this notation is used the terms ε^*_{t+1}, ε^F_{t+1}, μ, or MR_{t+1} also are (N,1) vectors.

These price dynamics imply at least 2 investment strategies : a long-term "portfolio" strategy based on fundamental drifts, a mid-term "convergence" one which exploits the short-term deviations from fundamental value. In this model each strategy is followed respectively by "portfolio investors", and "convergence traders". As an extension we introduce in section 5 "speculators" who follow a "short-term" strategy that focuses on the short term auto-regressive component of constrained selling embedded in $f(\varepsilon^*_t, \varepsilon^F_t)$.

The full specialization between long-term and mid-term investors results from an ex-ante arbitrage by both agents. Spotting non fundamental shocks on prices requires information and monitoring, for which convergence traders pay a fixed cost ϵ at each t. Such a market segmentation is close to that of Graham who differentiates an "active or enterprising approach to investing" from a "passive or defensive strategy that takes little time or effort but requires an almost ascetic detachment from the alluring hullabaloo of the market" (Zweig, 2003, p101).

Long-term investors hold a diversified portfolio, with $n \in |[1, N]|$ the level of diversification, and are subject to the regulatory constraint. The impact of diversification is therefore studied through how much of and what assets portfolio investors will sell. Initially M long-term investors of same type I are endowed with "their" asset i=I, so that they are $M\times N$ long term investors. Convergence traders and speculators respectively provide demand and additional supply for the assets sold. Both are segmented as in Merton (1987).

We introduce some notation: for a given investor z, $q_{i,z}$ represents the actual quantity of asset i, $q^*_{i,z}$ the desired one, $q_z = \Sigma_{i=1}^{i=N} q_i$ is his total investment in all risky assets. $q_i = \sum_{z=1}^{z=N} q_{i,z}$ is the total quantity of asset i across investors. Note also that in what follows we sometimes only refer to constrained *selling* by portfolio investors, because we are primarily interested in fire sales. However the model applies equivalently to a situation in which portfolio investors are *buyers*, as their constraint gets more loose following a positive shock on their wealth.

2.2 Investors

Both mid-term and long-term investors have CARA utility, with risk aversion of τ_{mt} and τ_{lt} respectively. The risk aversion is similar across investors of the same type, each agent is a price-taker. Both types base their strategies on assessing, correctly, the fundamental evolution of assets. Convergence traders need this knowledge to assess how current prices deviate from their fundamental values, portfolio investors need it to evaluate the long-term returns. They know μ and σ^F.

2.2.1 Mid-term investors

They hold assets during t^* periods, the time it takes for trading shocks to return to 0, and pay ϵ to be able to track prices at each t. They are free of regulation and have "deep pockets", so that $q^*_{i,t} = q_{i,t}$. Mathematically each z solves:

$$Max\, E(-e^{-\frac{w_{z,t+t^*}}{\tau_{mt}}})$$

$$u/c \;\; w_{z,t+t*} = w_{z,t} + q^*_{i,z,t}(p_{i,t+t*} - p_{i,t}) - t^*\epsilon$$

Using the moment generating function[3] yields the vector of optimal quantities for convergence traders:

$$q^{mt}_{i,z,t} = \tau_{mt}\frac{E(p_{i,t+t*} - p_{i,t})}{\sigma^2_{t+t^*}}$$

where $\sigma^2_{t+t^*}$ is the variance of asset i at horizon $t + t^*$, which is known by convergence traders.

When $t*$ is large, all trading shocks, whether they have occurred or are expected to, will vanish through mean-reversion. We thus have[4]:

$E(p_{i,t+t*} - p_{i,t}) = E(\mu t^* + \sum_{j=t+1}^{j=t+t*} MR_{i,j} + \sum_{j=t+1}^{j=t+t*} \varepsilon^F_{i,j} + \sum_{j=t+1}^{j=t+t*} \varepsilon^*_{i,j}) = \mu t^* + (p^F_{i,t} - p_{i,t})$

First differencing we obtain the demand/supply shock for a given convergence trader between t and *t+1*:

$$\triangle q^{mt}_{i,z,t} = \tau_{mt}\frac{(p^F_{i,t+1} - p_{i,t+1}) - (p^F_{i,t} - p_{i,t})}{\sigma^2_{t+t^*}}$$

Rearranging and setting to m the number of convergence traders operating on each market, we obtain total demand/supply shifts from convergence traders between *t+1* and t, for a given asset i :

$$\triangle q^{mt}_{i,t} = \frac{m\tau_{mt}[(\mu_t + \varepsilon^F_{i,t+1}) - (p_{i,t+1} - p_{i,t})]}{\sigma^2_{t+t^*}}$$

In matrix form:

$$\Longrightarrow \triangle \mathbf{Q^{mt}_t} = -h(MR_{t+1} + \varepsilon^*_{t+1}) \quad (3)$$

[3]See appendix A for a step by step derivation

[4]appendix B provides a step by step computation

where $h = \frac{m\tau_{mt}}{\sigma^2_{t+t^*}}$ represents the total increase in the quantity demanded in response to a rise in expected returns, and thus indicates the strength of the demand.

2.2.2 Long-term investors

Maximization problem

Portfolio investors time horizon is noted $t+T$. Formally:

$$Max\, E(-e^{-\frac{w_{z,t+T}}{\tau_{lt}}})$$

$$u/c \;\; w_{z,t+T} = w_{z,t} + \mathbf{q_t^*}^\intercal(\mathbf{P_{t+T}} - \mathbf{P_t})$$

As portfolio investors do not incur the monitoring cost ϵ, they can only estimate $(\mathbf{P_{t+T}} - \mathbf{P_t})$ through its long-term zero average.

$E(\mathbf{P_{t+T}} - \mathbf{P_t}) = E(\mathbf{P_t} + T\mu + \sum_{j=t+1}^{j=t+t*} MR_j + \sum_{j=t+1}^{j=t+T} \varepsilon_j^F + \sum_{j=t+1}^{j=t+T} \varepsilon_j^* - \mathbf{P_t})$
$= T\mu + \lambda E(\sum_{j=t+1}^{j=t+t*} (\mathbf{P_j^F} - \mathbf{P_j}))$
$= T\mu$

Contrary to the expected value of $\mathbf{P_t}$, the covariance matrix Σ_T does not move with each shock. It is correctly assessed. The vector of optimal quantities is therefore:

$$\mathbf{q_t^{*lt}} = \frac{\tau_{lt}\mu}{\Sigma_T}$$

The key element is that, as long as portfolio investors do not see a change in fundamental drift or variance, their demands will remain unaltered. Therefore with similar moments across assets, they ideally hold the same quantity of each of the asset featuring in his portfolio :

$$\frac{q_{i,z,t}}{q_{z,t}} = \frac{1}{n} \tag{4}$$

Regulatory constraint

Each portfolio investor is subject to a Cooke ratio type constraint: risk weighted assets over capital may not go above a given value α. Mathematically:

$$RWA_{z,t} \leq \alpha K_z \tag{5}$$

where $RWA_{z,t}$ is the risk weight on investor z, and K the capital level, which may differ across investors. In practice risk weights are discrete, here a continuous measure is needed. We use an inverse Sharpe ratio type measure[5] the ratio of variance over expected value, at horizon $t + t^*$:

$$RWA_{z,t} = \frac{Q_{z,t}^{lt} \sum_{t+t*} Q_{z,t}^{lt}}{E(P_{t+t*})Q_{z,t}^{p}}$$

[5] we discuss this choice in section 2.4

where $Q^{lt}_{z,t}$ is the vector of actual quantities held by z. The regulator's expectation of the evolution of prices is in line with that of portfolio investors: $P_{t+t*} = P_t + \mu t^*$. This will not hold for his estimation of the covariance between assets, as we assume the regulator refuses to take into account the variance reducing effect of diversification. Mathematically any element $a_{i,j}$ of $\sum_{t+t*}$ is $a_{i,j} = \sigma^2_{t+t^*}$, $\forall i$, $\forall j$.

Using (4) we may re-express both components of RWA :

$E(P_{t+t^*})Q^{lt}_{z,t} = \frac{q_{z,t}}{n}((t^*+1)n\mu + \sum_{i.} p_{i,z,t-1})$

where $\sum_i p_{i,z,t}$ represents the sum of the prices of assets that feature in z's portfolio, and $Q^{lt}_{z,t}\sum_{t+t*} Q^{lt}_{z,t} = q^2_{z,t}(\sigma^2_{t+t^*})$. For an investor z operating on his constraint we thus have:

$$\frac{q^2_{z,t}\sigma^2_{t+t^*}}{\frac{q_{z,t}}{n}((t^*+1)n\mu + \sum_{i.} p_{i,z,t-1})} = \alpha K_z \Longrightarrow q_{i,z,t} = \frac{1}{n}\frac{\alpha K_z(t^* n\mu + \sum_{i.} p_{i,z,t-1})/n}{\sigma^2_{t+t^*}}$$

The refusal by the regulator to factor in the gain in variance from diversification one of the well documented features of the Basel agreements (see Atkinson et al, 2010), it can be seen as a way for regulation put more weight on the undesirable outcomes. The fact that regulators monitor the health of long term investors at every t captures the pro-cyclical effect of marked-to-market based regulation, a major driver of fire sales according to Tirole (2008).

Note that we have used the expected value of the portfolio at *t-1* multiplied by quantity at *t*. This means if a long term investor is considered too risky at time *t-1*, he is required to lower his total investment in risky assets at *t*, so that the risk-weighted assets are below total equity at the beginning of the following period. Differencing and multiplying by the number of constrained investors of a given type I, noted δ, we obtain total sales of asset i by investors of type *I*, between *t-1* and *t*:

$$\triangle q_{i,I,t} = \frac{\delta\alpha K_I}{n\sigma^2_{t+t^*}}(\sum_i \triangle p_{i,I,t-1}/n)$$

$$\Longrightarrow \triangle q^{lt}_{i,I,t} = \frac{r}{n^2}\sum_i \triangle p_{i,I,t-1} \qquad (6)$$

where $r = \frac{\delta\alpha K_I}{\sigma^2_{t+t^*}}$ represents the selling of each asset i featuring in investor I 's portfolio following a shock, and thus represents the strength of the selling constraint. For conciseness, we now refer to this total selling by all investors of type I as the selling of representative "investor I" .

2.3 Network formation pattern, matrix form

The previous section showed how each representative investor I behaves. To study the systemic implications of these behaviors we must specify a pattern of asset holdings. Figure 1 summarizes the network formation: each holding of asset i by investor J is a connection between J and the investor I=i who initially held the asset. Investors are nodes and asset holdings are the connections between them. The numbers on each link thus list the assets that I and J have in common, indicating of how closely related they are.

As the degree of diversification n increases, each I acquires the asset that is the closest to his right. Then I_k holds:

- only asset k if $n = 1$, no diversification
- assets k and *k+1* if $n = 2$
-etc.

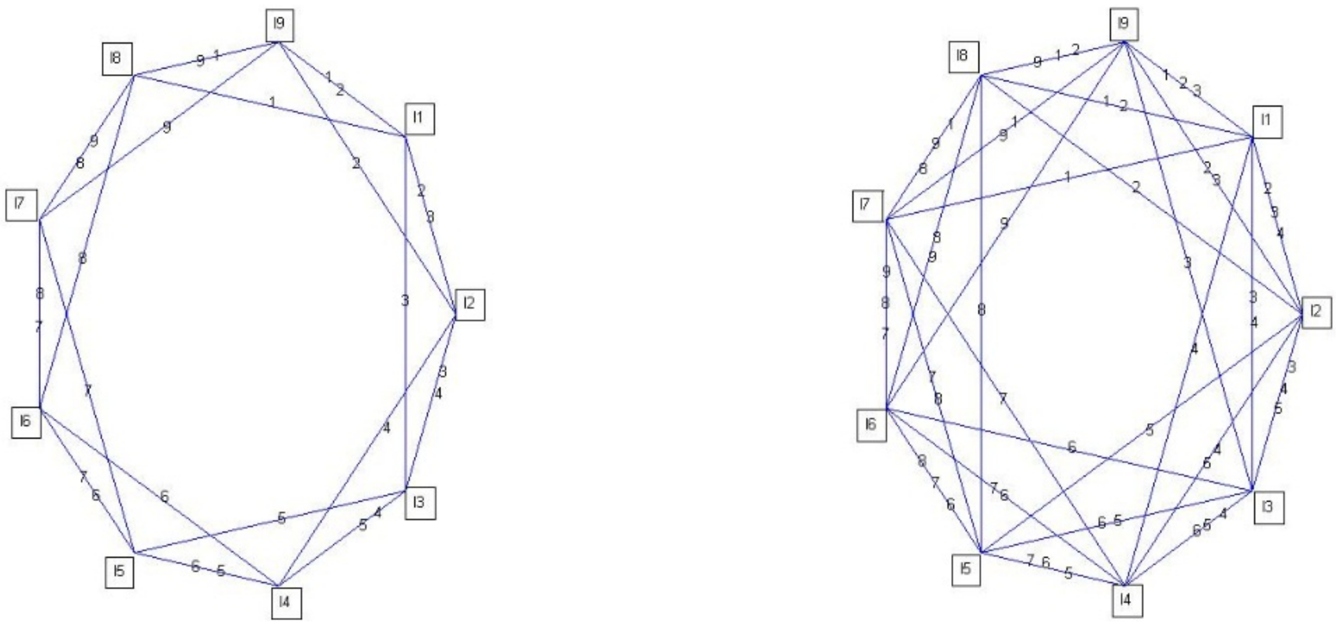

Figure 1: network formation and assets in common N=10, n=3, n=5

so that I_k holds assets k to k', where $k' \equiv k+n-1$ [N]

Although this network formation pattern is completely exogenous in our set-up, one could rationalize it using information costs, or any type of home bias.

This information may be expressed in matrix form by letting each row represent an investor I and each column an asset i, and setting *1* if I holds i , 0 otherwise. For instance if *N=5* and *n=3*:

$$T = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

vector of investors sales

The proportion of asset i in the portfolio of investor I is $1/n$ if i features in the portfolio, 0 otherwise. The average return on investor I 's portfolio thus writes:

$$\sum_i t_{I,i} \triangle p_{i,t}/n$$

And equation (6) may be re-expressed as:

$$\triangle q_{i,I,t} = \frac{r}{n^2} \sum_i t_{I,i} \triangle p_{i,t-1}$$

The vector of general element $\triangle q_{i,I,t}$ summarizing the unit selling by each long-term investor I as a response to a wealth shock, then writes:

$$\triangle \mathbf{q_t^{lt}} = \frac{r}{n^2} T \triangle \mathbf{P_{t-1}}$$

total selling per asset

Total selling for asset i is the sum of all selling of investors who holds the asset. The supply/demand shock between t and $t+1$ for a given asset i may then be expressed as

$$\triangle q_{i,t} = \sum_I t_{I,i} \triangle q_{i,I,t} = \sum_I t^{\top}_{i,I} \triangle q_{i,I,t}$$

where $t^{\top}_{i,I}$ is the general element of the transpose of T. As each row of T tells us what assets a given investor holds, each row of $T^{\intercal}$ indicates what investors hold a given asset i. Hence the expression of $\triangle \mathbf{Q_t^{lt}}$, the vector of general element $\triangle q_{i,t}$:

$$\triangle \mathbf{Q_t^{lt}} = T^{\intercal} \triangle \mathbf{q_t^{lt}}$$

$$\implies \triangle \mathbf{Q_t^{lt}} = \frac{r}{n^2} T^{\intercal} T \triangle \mathbf{P_{t-1}} \tag{7}$$

market clearing

Let us now take the market clearing condition $\triangle \mathbf{Q_t^{lt}} + \triangle \mathbf{Q_t^{mt}} + \triangle \mathbf{Q_t^{mr}} = 0$, where the terms on the right-hand side are the demand/supply shocks stemming from respectively portfolio investors, arbitrageurs, and the mean reversion to long term investment level. Substituting:

$$\varepsilon^*_{t+1} + MR_{t+1} = (\frac{r/h}{n^2}) T^{\intercal} T \triangle \mathbf{P_{t-1}} + \frac{\triangle \mathbf{Q_t^{mr}}}{h} \tag{8}$$

We give the model its final form by plugging (8) into the price dynamics equation:

$$\triangle \mathbf{P_t} = \mu + (\frac{r/h}{n^2}) T^{\intercal} T (\triangle \mathbf{P_{t-1}}) + \frac{\triangle \mathbf{Q_t^{mr}}}{h} + \varepsilon^F_{i,t+1} \tag{9}$$

The end product of the model is thus a stochastic recurrence system that maps how price shocks spread through time and assets. From this perspective, the fact that the matrix $T^{\intercal} T$ is circulant will come in handy.

Summary of the frame of the model:

- portfolio investor of type I hold assets i=I to i=I+n

- when faced with a negative shock I sells an equal quantity of every asset, where the amount sold depends on r, "the strength" of the constraint

- convergence traders buy these assets with a discount that depends on h, the "strength" of demand

- The market clearing condition then yields $\triangle \mathbf{P_t} = \mu + (\frac{r/h}{n^2}) T^{\intercal} T (\triangle \mathbf{P_{t-1}}) + \frac{\triangle \mathbf{Q_t^{mr}}}{h} + \varepsilon^F_{i,t+1}$

2.4 Assumptions

Before solving the model, it is appropriate to discuss 3 assumptions underlying it:

1) *Stock prices are normally distributed.* The relevance of this postulate have already been discussed extensively (Andersen et al., 2001): while admittedly a poor description of reality, its merit is to be much easier to manipulate then alternative distributions. We have also chosen normality over log-normality, as the economic logic of the model dictates we work with price evolution rather than returns.

Nonetheless, perhaps the most common criticism attached to normal price returns is their incapacity to capture the actual thickness of the tails. Our model shows that under a particular set of circumstances the system endogenously generates a dependence structure between assets which make such "fat tails" appear. The paper thus avoids this pitfall.

2) *Agents follow a linear rule.* On the supply side, this principle translates into the assumption that the regulator uses the inverse Sharpe ratio as weighting scheme. This method differs from common practice amongst academics and practitioners, which uses value-at-risk (see for instance Danielsson et al., 2011). Linearity also comes from the fact that agents correctly assess asset moments, and never question this assessment. In particular, convergence traders always provide demand with the same discount h, regardless of the current market conditions. This is consistent with the efficient market hypothesis, but arguably unrealistic. In practice increased risk aversion, or higher short-termism may hamper demand during crises. We account for such heuristics in section 4 by allowing risk aversion to rise in response to high selling movements.

3) *The regulator and arbitrageurs have the same mid-term horizon.* With a mismatch between both horizons the expression of the price vector and hence the covariance matrix would have depended on the covariance matrix itself, ie the study of the covariance would involve solving a fixed-point problem. Though they may be an important carrier of systemic risk, horizon mismatches are not the object of this paper. Bearing in mind the increase mathematical loading they would involve, we decided to align horizons.

4) *Network formation.* This choice was primarily motivated by economic intuition, as we wanted to account for home biases. However one should note that to obtain a circulant network we only require that all investors follow the same pattern, not necessarily a biased one. We analyze a bias-free network in section 5.

3 Endogenous correlations

To convey maximum intuition while minimizing quantitative heavy lifting, we will study how a single stochastic shock spreads through time. We thus set all shocks to 0 past $t{=}0$ for all securities, solving a deterministic version of the model, in which the total losses will depend on the realization of the stochastic initial price fall.

We also set the mean-reversion and fundamental drift to 0, for three reasons: a) our time unit is a "day" in the market, so that the fundamental long-term trends should be negligible anyway, b) they would overload the equations as a recurrence system with a drift is slightly more demanding and c) without changing the insights of our model, since the variance structure would not be changed by the inclusion of MR and μ.

We thus study the following system:

$$\triangle \mathbf{P_t} = (\frac{r/h}{n^2}) T^\intercal T \triangle \mathbf{P_{t-1}} \tag{10}$$

$$\Longrightarrow \triangle \mathbf{P_t} = [(\frac{r/h}{n^2}) T^\intercal T]^t \triangle \mathbf{P_0} \tag{11}$$

The study of how the system reacts to a fundamental shock involves finding $D_{T^\intercal T} = diag(\phi_0, .., \phi_{N-1})$, the diagonal matrix similar to $T^{\intercal}T$, and its eigenvalues ϕ_i .

3.1 Diagonalizing $T^{\intercal}T$

From the cyclic permutation matrix J to T

Let J be the "cyclic permutation matrix", whose element $a_{i,j} = 1$ if $i \equiv j - 1$ [N], $a_{i,j} = 0$ otherwise. For instance if *N=5*:

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Taking J to the power n shifts the one-diagonal *n-1* spots to the right. For instance :

$$J^2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvalues of J are readily obtained from the observation that $J^N = 1^N$. Using the Cayley-Hamilton theorem, the characteristic polynomial of J is of the form $Q[x] = x^n - 1$, and its eigenvalues are the nth-roots of unity. Thus the matrix similar to J is $D_J = diag(\omega^0, \omega, ..., \omega^{N-1})$ where $\omega^k = e^{\frac{2i\pi k}{N}} = cos(\frac{2\pi k}{N}) + i\ sin(\frac{2\pi k}{N})$ and $\omega^{-k} = cos(\frac{2\pi k}{N}) - i\ sin(\frac{2\pi k}{N})$ according to Euler's identity. We thus have $J = PD_JP^{-1}$, where P is the change of basis matrix. Note also that since $\omega^N = 1$, we have $\omega^{\alpha(N-k)} = \omega^{\alpha N - \alpha k} = \omega^{-\alpha k}$.

From the properties of J, any circulant matrix C may therefore be expressed as a polynomial in $J : C = R(J)$. This implies $C = R(PD_JP^{-1}) = PR(D_J)P^{-1}$, so that the matrix similar to any circulant matrix is a polynomial of D_J, and P is the change of basis matrix for all circulant matrices. In particular for T:

$$T = R(J) = \sum_{s=0}^{s=n-1} J^s$$

where n is the level of diversification. The general expression of T's eigenvalue ψ_k is therefore the inverse Fourier transform $\psi_k = R(\omega_k) = \sum_{s=0}^{s=n-1} \omega^{ks}$, which can be expressed as the sum a geometric series of n terms and common ratio $e^{\frac{2i\pi k}{N}}$, so that

$$\psi_k = \frac{1 - \omega^{kn}}{1 - \omega^k} = \frac{1 - cos(\frac{2\pi kn}{N}) - i\ sin(\frac{2\pi kn}{N})}{1 - cos(\frac{2\pi k}{N}) - i\ sin(\frac{2\pi k}{N})} \tag{12}$$

except when *k=0*, in which case $\psi_0 = n$. Note that this expression also implies $\psi_{-k} = \psi_{N-k}$.

From T to $T^{\intercal}T$

For this step we must uncover the change of basis matrix *P*. $JP = PD_j$ yields $p_{i+1,j+1} = (\omega^j)^i p_{1,j}$, etc. Setting

$p_{1,z} = \frac{1}{\sqrt{N}}$ we obtain an orthonormal base for the eigenvectors of the discrete inverse Fourier transform matrix of coefficient $\frac{1}{\sqrt{N}}$, whose general term is $p_{i+1,j+1} = \frac{(\omega^j)^i}{\sqrt{N}}$, with $(i,j) \in \{0,1,..,N-1\}^2$:

$$P = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & . \; . & 1 \\ 1 & \omega & . \; . & \omega^{N-1} \\ . & . & & . \\ . & . & & . \\ 1 & \omega^{N-1} & & \omega^{(N-1)^2} \end{pmatrix}$$

P has numerous useful properties:

1) it is unitary: its inverse is equal to its conjugate transpose. $P^{-1} = P^{\dagger}$, whose general term is $\frac{\omega^{-jk}}{\sqrt{N}}$

2) $P = P^{\intercal}$ and $P^{-1} = (P^{-1})^{\intercal}$, so that $T^{\intercal} = P^{-1} D_T P$

3) The square of P and P^{-1} is a near reversal matrix noted V. For instance if N=5:

$$V = P^2 = P^{-2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

We may then write $T^{\intercal}T = P^{-1}D_T P P D_T P^{-1} = P^{-1} D_T V D_T P^{-1}$. Pre-multiplying by PP^{-1} we get the desired result:

$$T^{\intercal}T = P(VD_T)^2 P^{-1} \tag{13}$$

where $(VD_T)^2$ is the diagonal matrix similar to $T^{\intercal}T$. We may then re-express equation (11):

$$\triangle \mathbf{P_t} = (\frac{r/h}{n^2})^t P (VD_T)^{2t} P^{-1} \triangle \mathbf{P_0} \tag{14}$$

$\triangle \mathbf{P_t}$

General element of $\triangle P_t$

The element of $(VD_T)^2$ on row $k+1 > 1$ may be expressed[6] as $\phi_k = \phi_{N-k} = \psi_k \psi_{-k} = \frac{1-\omega^{kn}}{1-\omega^k} \times \frac{1-\omega^{-kn}}{1-\omega^{-k}}$, which will be positive for any k. Using (12) and rearranging we obtain:

$$\phi_k = \phi_{N-k} = \frac{1 - cos(\frac{2\pi kn}{N})}{1 - cos(\frac{2\pi k}{N})}$$

If k=0, $\phi_0 = n^2$.

(14) then yields the general expression of the evolution of the price of a given security k[7] :

[6]see appendix C for a derivation through an N=4 example

[7]see appendix D for a derivation through an N=4 example

$$\triangle p_{k,t} = \frac{1}{N}\sum_{j=0}^{j=N-1}\triangle p_{0,j}\sum_{q=0}^{q=N-1}\omega^{(k-j)q}(\phi_q)^t \quad (15)$$

Using Euler's identity, we have $\omega^{(k-j)(N-q)}\phi_{N-k} + \omega^{(k-j)q}\phi_k = 2cos(\frac{2i\pi(k-j)q}{N})\phi_k$, we may then express the evolution of a given asset k's price over time as:

$$\triangle p_t = (\frac{r/h}{n^2})^t \frac{1}{N}\sum_{j=0}^{j=N-1}\triangle p_{0,j}[n + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)^t] \quad (16)$$

when N is odd [8].

Asset k depends on the evolution of all the other assets of the economy. The impact of each asset i on k depends on the distance i-k in the financial network, the conditions on the market r/h, the completeness of the market N, and of course the level of diversification n.

Convergence of the system

The system will converge if all the elements of $\frac{r/h}{n^2}{D_T}^{2t}$ have a modulus below one. The maximum eigenvalue of $T^\intercal T$ is $|\psi_0| = (\sum_{s=0}^{s=n-1}\omega^0)^2 = n^2$. Therefore the convergence condition for $\frac{r/h}{n^2}{D_T}^{2t}$ is $\frac{r/h}{n^2}n^2 < 1$ or:

$$r < h$$

In other words, the increase in demand following a drop in prices must be superior to that of constrained supply, otherwise prices are ever-falling. This condition is necessary to study the distribution of the vector of price dynamics to be well-behaved, as if $r/h > 1$, we would have $lim_{t\to+\infty}var(\triangle P_{j,t}) = +\infty$.

Summary of the diagonalization of $T^\intercal T$:

- Matrix J's eigenvalues are the roots of unity

- Matrix T is a polynomial of J, so its eigenvalues are a polynomial of J's eigenvalues

- All circulant matrices have the change of basis matrix P where $PP = V$, V a near reversal matrix

- This implies $T^\intercal T = P^{-1}D_T V D_T P^{-1}$, of general element given by (16)

3.2 Endogenous correlations

We study the distributions of the vectors $\triangle P_t$, $\Sigma_{t=0}^{t=+\infty}(\triangle P_t)$, and $\frac{1}{n}T\Sigma_{t=0}^{t=+\infty}(\triangle P_t)$, respectively price changes at a given period t, across time, and portfolio return for each investor across time. All three vectors are a linear combination of the initial normally distributed shocks, so they are normally distributed. Each stochastic vector's expected value is 0, we only need to find the realized covariance matrices to obtain the distributions.

In assets per period

The covariance matrix at a given t is $\sum_t = \sum_t = E([\triangle P_t - E(\triangle P_t)][\triangle P_t - E(\triangle P_t)]^\intercal) = E(\triangle P_t {\triangle P_t}^\intercal)$, since $E(\triangle P_t) = 0$. Using (14) and the fact that $\triangle P_0 = \varepsilon_0^F$ we re-express:

[8] $(\frac{r/h}{n^2})^t\frac{1}{N}\sum_{j=0}^{j=N-1}\triangle p_{0,j}(2\sum_{q=1}^{q=N/2-1}cos(\frac{2\pi(k-j)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)^t + n^{2t} + cos(\pi(k-j))\left(\frac{1-cos(n\pi)}{2}\right)^t)$ if N is even

$$\sum_t = (\frac{r/h}{n^2})^{2t} P(VD_T)^{2t} P^{-1} E(\varepsilon_0^F \varepsilon_0^{F\intercal}) P^{-1} (VD_T)^{2t} P \tag{17}$$

where $\varepsilon_0^F \varepsilon_0^{F\intercal}$ is the fundamental covariance matrix, and $E(\varepsilon_0^F \varepsilon_0^{F\intercal}) = diag(\sigma_F^2)$. Therefore:

$$\Sigma_t = \sigma_F^2 (\frac{r/h}{n^2})^{2t} P(VD_T)^{2t} V (VD_T)^{2t} P \tag{18}$$

The matrix is symmetric since $V = V^\intercal$ and $P = P^\intercal$. Post-multiplying by PP^{-1} we find : $\Sigma_t = \sigma_F^2 (\frac{hr}{n^2})^{2t} P(VD_T)^{2t} V (VD_T)^{2t} V P^{-1}$, so that $(VD_T)^{2t} V (VD_T)^{2t} V$ is the matrix similar to Σ_t. It is diagonal, and its eigenvalues are positive, hence Σ_t is definite positive.

The general expression of the covariance[9] between two securities k and j writes[10] :

$$cov(\triangle p_j, \triangle p_k)_t = \frac{\sigma_F^2}{N} (\frac{r/h}{n^2})^{2t} (2 \sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N}) \left(\frac{1 - cos(\frac{2\pi nq}{N})}{1 - cos(\frac{2\pi q}{N})} \right)^{2t} + n^{4t})$$

This result is in line with the growing literature on endogenous risk: the actions of the market participants lead to correlations between assets that would not exist otherwise. One of the contributions of the paper is to make these correlations contingent on the nature of the links between investors who hold them in the financial network.

n enters negatively through the $(\frac{r/h}{n^2})^{2t}$ term, and positively through the general element of $P(VD_T)^{2t}V(VD_T)^{2t}P$. This reflects a dual effect effect of diversification on covariances. On one hand diversification immediately creates bridges between assets, on the other in a diversified economy shocks die out more quickly , so that covariances will converge to 0 rapidly. For instance moving from n=*2* to n=*3* involves a higher covariance between i and i+*2* at t=*1*, maybe at t=*2,* but probably not in subsequent periods as $(\frac{r/h}{n^2})^{2t}$ starts dominating. The impact of diversification on the total transmission between assets i and i+*2* is thus ambiguous.

However for assets which are already direct neighbors such as i and i+*1* in our example, further diversifying unambiguously lowers their covariances.

In assets across time

Let us first define the vector :

$$\Sigma_{t=0}^{t=+\infty} (\triangle P_t - E(\triangle P_t)) = P(\sum_{t=0}^{t=+\infty} (\frac{r/h}{n^2})^t (VD_T)^{2t}) P^{-1} \varepsilon_0^F = P D_{tot} P^{-1} \varepsilon_0^F$$

where the element of row k of $D_{tot} = \Sigma_{t=0}^{t=+\infty} [(\frac{r/h}{n^2})^t (VD_T)^{2t}]$ is $\xi_k = \Sigma_{t=0}^{t=+\infty} (\frac{r/h}{n^2} \phi_k)^t$, the sum of a geometric series of common ratio $(\frac{r/h}{n^2} \phi_k)$, whose value is below one since r/h<1. Letting the number of periods tend to infinity:

$$\xi_k = \frac{1}{1 - (\frac{r/h}{n^2} \phi_k)} = \frac{1}{1 - (\frac{r/h}{n^2} \frac{1 - cos(\frac{2\pi kn}{N})}{1 - cos(\frac{2\pi k}{N})})} \tag{19}$$

with $\xi_0 = \frac{1}{1 - r/h}$ and $\xi_k = \xi_{N-k}$ since $\phi_0 = n^2$ and $\phi_k = \phi_{N-k}$.

The ex post covariance matrix is therefore:

[9] see appendix D for a derivation through an N=*4* example

[10] If N is even $\frac{\sigma_F^2}{N} (\frac{r/h}{n^2})^{2t} (2 \sum_{q=1}^{q=(N/2-1)} cos(\frac{2\pi(k-j)q}{N}) \left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})} \right)^{2t} + n^{4t} + cos(\pi q) \left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})} \right)^{2t})$ If N is even

$$\Sigma_{tot} = E((PD_{tot}P^{-1}\varepsilon_0^F)(PD_{tot}P^{-1}\varepsilon_0^F)^\intercal) = \sigma_F^2 PD_{tot}VD_{tot}P$$

We obtain the general element of the covariance between two assets k and j[11] :

$$cov(\triangle p_j, \triangle p_k)_{tot} = \frac{\sigma_F^2}{N}((\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2) \tag{20}$$

The exact impact of diversification on the covariances still depends upon the parameters. Running simulations[12] suggests that r/h is particularly important. For back to our example of i and $i+2$ when n moves from 2 to 3, we find that the covariance may be higher when $n=2$ for high values values of r/h. In other words the fact that the shock dies out more quickly when spread across assets more than compensates the immediate increased contagion at t=1. This is logical, when shocks are very persistent spreading them becomes more desirable.

Investors covariance

We label investors covariance the covariance between the average return of portfolios across long-term investors. The vector of per period average price return may be written as $I_t = \frac{1}{n}T\triangle P_t$, and the total price return over the period, on which we shall focus, is $I = \frac{1}{n}T\sum_{t=0}^{t=+\infty}\triangle P_t$. The covariance matrix thus writes: :

$$E([\frac{1}{n}T(\Sigma_{t=0}^{t=+\infty}(\triangle P_t - E(\triangle P_t))][\frac{1}{n}T(\Sigma_{t=0}^{t=+\infty}(\triangle P_t - E(\triangle P_t))]^\intercal) = \frac{\sigma_F^2}{n^2}TPD_{tot}VD_{tot}PT^\intercal$$

$$= \frac{\sigma_F^2}{n^2}PD_TP^{-1}PD_{tot}VD_{tot}PP^{-1}D_TP = \frac{\sigma_F^2}{n^2}P(D_TD_{tot})V(D_{tot}D_T)P$$

$$\Sigma_I = \frac{\sigma_F^2}{n^2}P(D_TD_{tot})V(D_{tot}D_T)P$$

The general element of the covariance between two investors k and j [13]:

$$cov(I_k, I_j)_{tot} = \frac{\sigma_F^2}{N}\frac{1}{n^2}[(n\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})\left((\frac{1-cos(\frac{2\pi kn}{N}) - i\,sin(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N}) - i\,sin(\frac{2\pi k}{N})})(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})\right)^2] \tag{21}$$

Similar to the assets case, the relationship between n and the covariances will depend on the parameters. Running simulations we also find that the covariance between total portfolio between two investors I and $I+2$ value

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[11] $\frac{\sigma_F^2}{N}((\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2 + cos(\pi q)(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2)$ if N is even

[12] available on request

[13] $\frac{\sigma_F^2}{N}\frac{1}{n^2}((n\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=N/2-1} cos(\frac{2\pi(N/2)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2 + cos(\frac{2\pi(N/2)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)(\frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2$ if N is even

fall may be higher for *n=2* then *n=3,* ie higher when they are not directly linked in the financial network. To see how this covariance influence systemic risk, ie the risk that many or all investors fall, we now move a statistical analysis.

4 Multivariate distributions

We discuss the impact of diversification on the likely number of bankruptcies for different parameter sets, and specify an objective function for society in which the cost is an exponential function of the number of failures. We only study the distribution of investors portfolios, leaving an analysis of asset prices contagion to section 5.

4.1 Baseline model

4.1.1 Choice of parameters

Prices and quantities are normalized to 1. Results will depend on 5 parameters: the fundamental variance σ^2, the maximum loss K that investors can incur before bankruptcy, the number of assets *N*, and finally the conditions on the markets *r/h*. We choose our baseline parameter set to fit the state of the financial markets coming in to the 2008 credit crunch. We normalize prices and initial quantities at *t=0* to 1, which does not impact our results.

To estimate the fundamental variance we compute the average daily variance of an asset belonging to the S&P 500, from the first of July 1997 to the first of July 2007. This yields $\sigma^2 = 0.000616$. The S&P was chosen because it is designed to provide a broader description of the investment opportunities than its counterparts, and equities are the asset class which suits our model the most. In our model noise is not only fundamental during crisis. Therefore period of measurement is set to avoid the subprime crisis but includes the internet bubble, which we considered as an increased volatility episode rather than a systemic event.

An investor goes bankrupt when his losses exceed his capital K. We use data of the World Bank to set *K=0.08*, that is 8% of normalized assets. This value stands between the reported capital ratios of US banks of 9,1% in 2008, and that of European ones, usually around 5%. We have somewhat arbitrarily chosen to be on the high side, to reflect the higher weight of the US markets.

N represents the number of assets in our model but in essence describes more the number an asset classes. Anecdotal evidence from investment funds implies a number of asset between 5 and 15. Furthermore, seeing *N* as the maximum diversification level use, we may follow a classic paper from Evans and Archer (1968), who estimate that diversification is no longer profitable past 10 securities, a belief shared amongst practitioners. We thus set N=10

The parameters mentioned have fairly straightforward implications: a more risky portfolio or a lower default threshold makes each investor marginally riskier, while increasing the number of asset decreases the likelihood attached to any possible number of bankruptcies in a uniform manner. For conciseness we thus keep these parameters constant throughout the paper. Multivariate distributions with different values for *K'* , σ^2, and *N* are available on request.

With respect to market conditions r/h, we study 3 scenarios:

- a "mild" one in which the constrained agents are forced to sell assets in fairly small quantities when a shock hits, and the demand by active investors of such assets is strong. $r/h = 0.6$

- a "windy" scenario in which the quantities sold by constrained agents and the demand by active investors is are both moderate . $r/h = 0.75$

- a "storm" scenario in which the pro-cyclical effect of regulation is large, and the response by active investors is weak. $r/h = 0.9$

Studying 3 scenarios results primarily from the lack of data on fire sales by investors[14], but is also interesting as part of our analysis.

4.1.2 Distribution of total number of bankruptcy

Figure 3 shows the likelihood attached to any number of investors falling, from 0 to N=*10*, for each level of diversification, in the r/h=*0.75* case.

This figure summarizes our findings and methods. Each possible number of investor failures has a probability given by the density function, and this probability varies with the level of diversification. The dual impact of diversification appears clearly. As n rises investors become more and more dependent, outcomes in which some investors fail but other survive become less likely. In the total diversification case, only the "all survive" and the "all fail" outcomes are possible. Due to the individually risk-reducing impact of diversification, the "all survive" equilibrium increases faster in likelihood, yet we also observe a gradual detachment from 0 of the likelihood that every investor fail, reaching a non-trivial 0.005% for n=*10.* In this case the overall desirability is thus ambiguous, as it will depend on how painful mass failure is to the entire economy.

Figures 4,5,6 zoom on number of failures large enough to constitute a systemic event, with r/h values of 0.6, 0.75, 0.9 respectively.

In figure 3 we see that any number of failure above 6 has a near zero likelihood. For low levels of diversification this result mainly from the higher independence between investors. For high levels of diversification it comes from the fact that with r/h=*0.6* shocks quickly die out, so that the contagion externality is limited. In figure 4 the scope for contagion is increased as we move to $r/h = 0.75$, this leads the probability of mass failure to become non-trivial. In the extreme r/h=*0.9* case of figure 5 shocks transmit almost fully across investors, so that the probability of mass failure, particularly the "all-fail" outcome reaches up to 15% in when n=N. In this case promoting an increased independence amongst investors seems sensible.

The dynamics of the rise in the odds of the all-fail outcome are also interesting. In the r/h=*0.6* case this likelihood when n=*3* is only 1% of that when n=N, against 7% if r/h=*0.75* and 41% in the r/h=*0.9* case. The marginal impact of a rise in n is thus increasing in calm market conditions, but decreasing in adverse ones.

4.1.3 Welfare

After discussing diversification's impact on the probabilities attached to any number of failures, we now weight such probabilities against the cost attached to each event.

If the cost to society increased in a linear fashion with the number of failures, diversification would unambiguously be desirable from society's perspective. Yet there are many theoretical reasons for which this cost may in fact grow exponentially with the numbers of defaults. In particular, the well-identified channels for contagion may be enhanced by bankruptcies. For instance, we expect surviving investors to become much more risk averse, reputation risk to sky-rock, the liquidity constraint to tighten, etc. Bernanke (1983) also highlights that financial bankruptcies have a

[14] There has been empirical evidence on the presence of fire sales (see for instance Coval and Stafford (2007)), but it is hard to use it for calibration as researchers can only conjecture that a given sale has been made out of necessity, and their impact on prices.

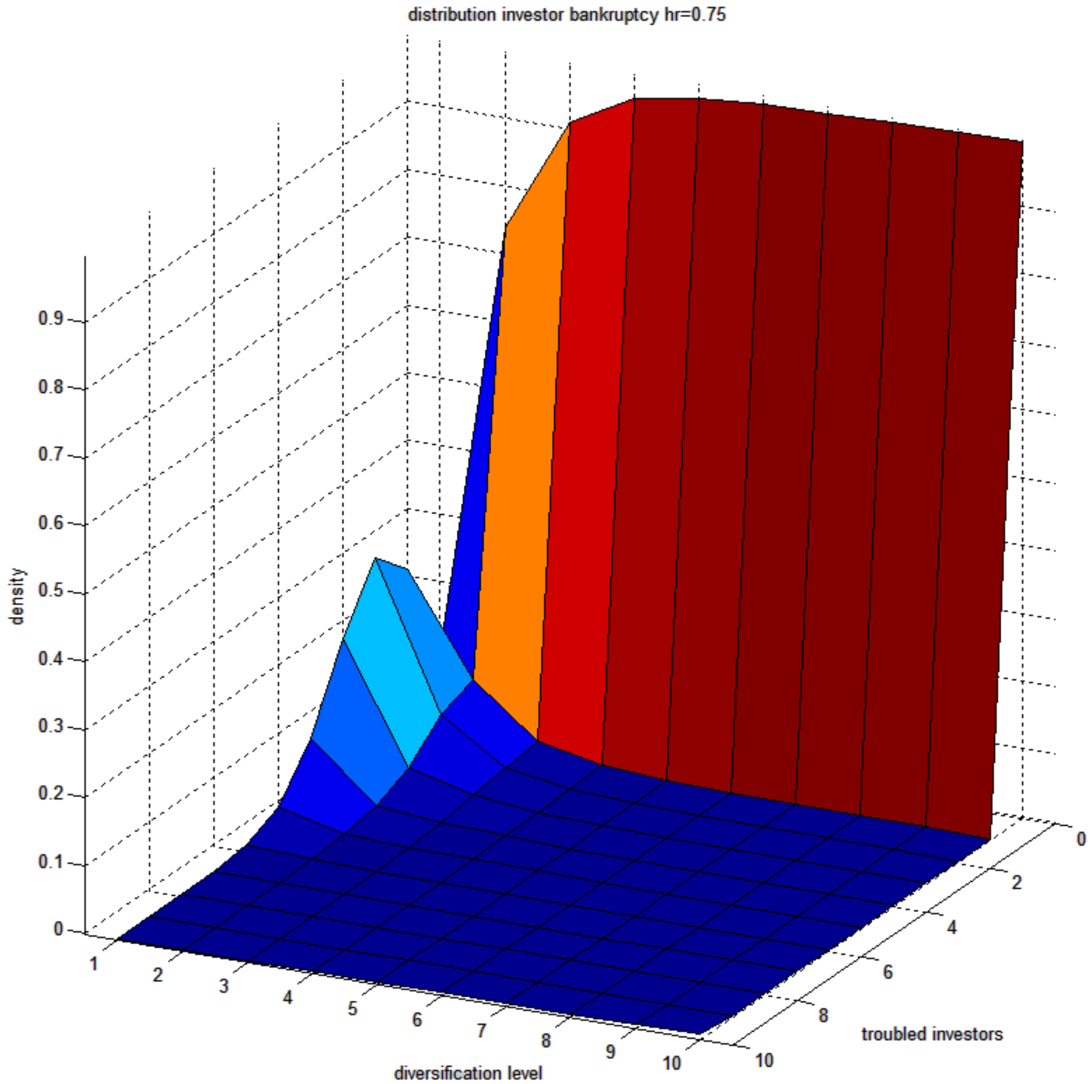

Figure 2: distribution of number of bankruptcy in stable regime with *r/h=0.75*

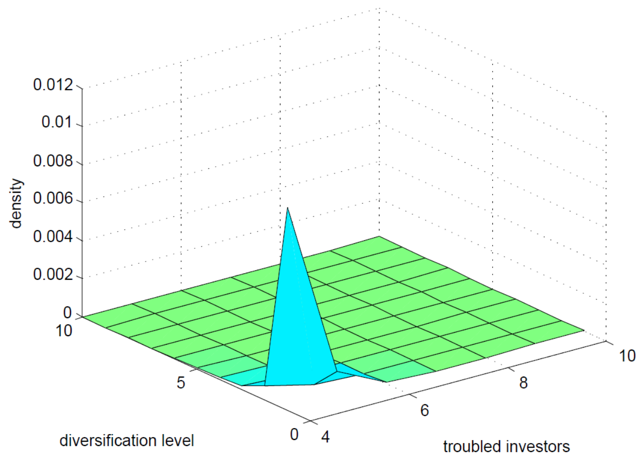

Figure 3: extreme bankruptcies odds, r/h=*0.6*

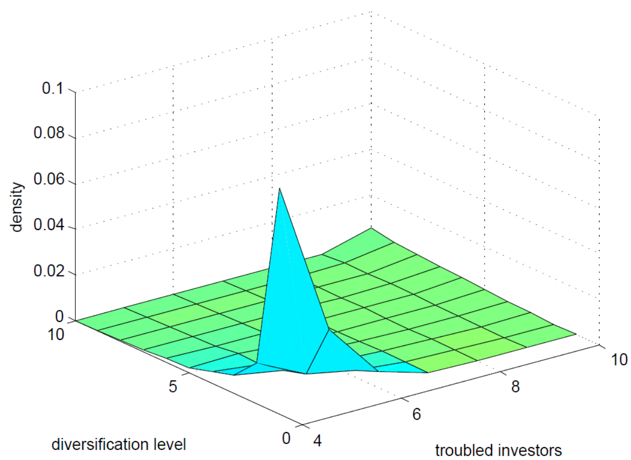

Figure 4: extreme bankruptcies odds, r/h=*0.75*

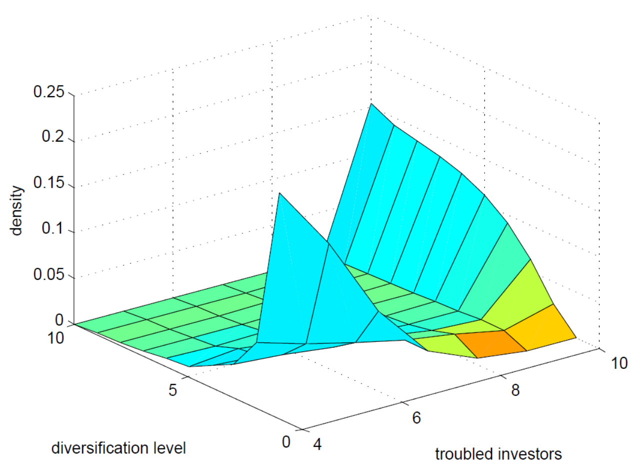

Figure 5: extreme bankruptcies odds, r/h=*0.9*

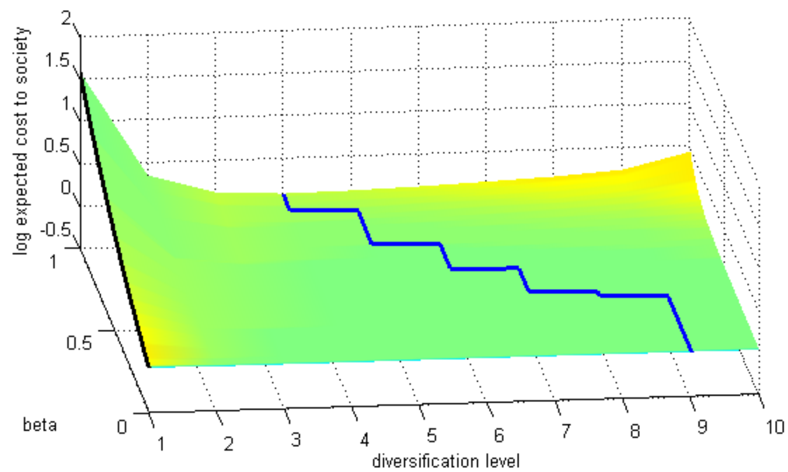

Figure 6: desirability of diversification with *r/h=0.6*

more than proportional impact on the real economy, through decreased money supply and increased cost of financial intermediation.

Perhaps due to this high variety of channels and non-linearity, there are to our knowledge no estimates of the exact cost to society of financial bankruptcies, and authors who want to model this cost have used different mathematical artifices. For instance Ibrahimov et al. define a time to recovery for the system, which depends of the number of defaults. We simply specify the following cost function for society:

$$C(\eta) = e^{\beta\eta}$$

where η, the number of failures, is a random variable, and β mitigates the severity of the increase in the cost to society of an additional failure. We show results for values of $\beta \in]0,1]$. To get an idea of the magnitude of the cost increase with the number of failures η: with $r/h = 0.75$, $\beta = 1$ implies an average cost of a single bankruptcy is 810 times larger when $\eta = N$ than it is if $\eta = 1$.

The expected cost to society writes:

$$E(C) = \sum_{\eta=0}^{\eta=N} P(\eta)C(\eta)$$

Where the probabilities $P(\eta = i)$ have been computed in last section, and of course depend on the level of diversification in the economy. The next 3 figures show this expected cost for our three values of *r/h*, across β and n. The blue line tracks the level of diversification in which the expected cost to society for a given $(r/h, \beta)$ couple in the lowest, the black line is the highest.

We find, as expected, that the expected systemic cost rises with β, and that a higher beta unambiguously works

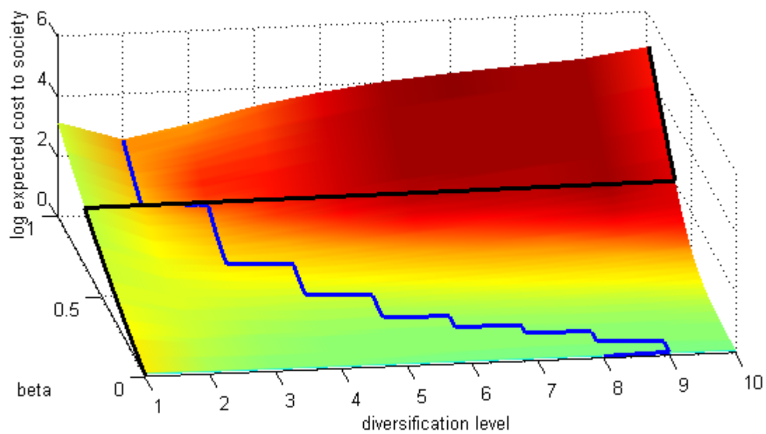

Figure 7: desirability of diversification with *r/h=0.75*

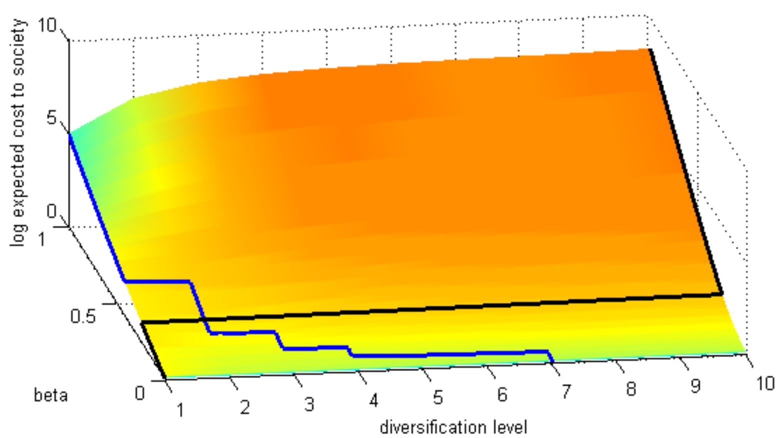

Figure 8: desirability of diversification with *r/h=0.9*

against higher levels of diversification, in which the number of failures is higher conditional of one bankruptcy. Graphically this is shown by the left turns of the blue line plotting the optimal diversification level.

Individually, the impression left from the previous section remains. In the *r/h=0.6* case the contagion externality is too modest for diversification not to be desirable. However with large value of β the optimal level of diversification goes surprisingly low, reaching *n=4*. In *r/h=0.75* the same logic applies, leading the optimal level to *n=2*, the point at which the risk of a large number of failures, and in particular the all-fail outcome, is still acceptable in exchange for the huge private benefits of moving from n=1 to n=2. The contagion externality is larger, leading n=N to become the least desirable level for $\beta > 0.7$. In the last *r/h=0.9* situation, the scope for contagion is so high that no individual risk reduction justifies it past $\beta = 0.4$. The perfectly diversified situation becomes the worst possible situation rapidly, when $\beta > 0.2$.

$\beta = 0.7$ implies that the ratio of unit of failure in the "all fail" over that in the "one only" case is 54.5, while $\beta = 0.4$ implies a ratio of 3.65. As mentioned this measure is based more on intuition than evidence, we leave it to the reader to assess where its true value would stand. In any case its seems fair to say that, in a situation in which financial shocks propagates linearly, there exist a reasonable set of parameters in which any level of diversification in the economy is dominated by a situation in which investors only trade only their own assets, and complete diversification is generally not the optimal level for society.

4.2 With possible panic

4.2.1 Change in framework

Remember the demand response to a deviation from fundamentals by convergence trader is given by h, where $h = \frac{m\tau_{mt}}{t*\sigma_F^2+\sigma_C^2}$, and $1/\tau_{mt}$ represents risk aversion. We now allow this risk aversion to change in the face of extreme selling movement on the markets. This could happen if for instance convergence traders no longer had deep pockets, and cared about their short-term ability to absorb losses as well as their mid-term benefits. We merely capture it though an exogenous rise in $1/\tau_{mt}$, which involves a fall in h and a rise in r/h. The system now writes:

$$\triangle \mathbf{P_t} = (\frac{r/h*}{n^2} T^\intercal T)^t \varepsilon_0 \quad if \; \exists \triangle q_i, \mid \triangle q_i \mid > k*$$

$$\triangle \mathbf{P_t} = (\frac{r/h}{n^2} T^\intercal T)^t \varepsilon_0 \quad if \; \forall \triangle q_i, \mid \triangle q_i \mid \leq k*$$

Where $h*$ is the discount prevailing in a situation of "panic". Therefore in this frame work investors "panic" when constrained sales get passed a certain threshold on a given market. It may thus only take one extreme movement on one market to trigger panic. Both experience and theory justify this form: increased risk aversion is highly contagious so that investors operating on the troubled market may lead others to grow more risk averse, accrued counterpart uncertainty is higher when one asset falls drastically than when all fall moderately, margin calls may be triggered when losses exceed a certain level, etc.

We note α the probability that one or more market reaches the sales threshold. It is given by the multivariate cumulative normal distribution Φ since sales depend linearly on the price shocks at *t=0* , which are normally distributed. We thus have $1 - \alpha = prob(\forall \triangle q_i, \mid \triangle q_i \mid \leq k*) = \Phi(k*, ..., k*)$. The expected value and variance of the sales vector at *t=0* are respectively 0 and $\Sigma_{\triangle Q} = E((\frac{r}{n^2} T^\intercal T \triangle P_0)(\frac{r}{n^2} T^\intercal T \triangle P_0)^\intercal) = \sigma_F^2 (\frac{r}{n^2})^2 P (VD_T)^2 V (VD_T)^2 P$.

Before proceeding, two things to note here. First this technique is consistent with our set-up since the sales are the largest at *t=0*. This means the threshold is either immediately reached or never in our framework, but in both cases $r/h*$ of r/h remains constant afterward. In other words the initial sales shock "sets the tone" for the rest of the crisis episode. Second though we focus on supply shocks by constrained investors, ie fire sales, the effect works symmetrically in the opposite direction, a unexpectedly large demand shock will lead convergence traders to feel more confident.

Mathematically this means the expected value of $\triangle\mathbf{P_t}$ remains zero regardless of the regime, and its variance [15]is given by:

$$V(\triangle\mathbf{P_t}) = [(1-\alpha)(\frac{hr}{n^2})^{2t} + \alpha(\frac{hr*}{n^2})^{2t}]\sigma_F^2 P(VD_T)^{2t}V(VD_T)^{2t}P \quad (22)$$

Which implies the following variances for assets and investors over time respectively:

$$V(\sum_t \triangle\mathbf{P_t}) = \sigma_F^2[\alpha PD_{tot}^* VD_{tot}^* P + (1-\alpha)PD_{tot}VD_{tot}P]$$

$$\Sigma_I = \frac{\sigma_F^2}{n^2}[\alpha P(D_T D_{tot}^*)V(D_{tot}^* D_T)P + (1-\alpha)P(D_T D_{tot})V(D_{tot}D_T)P] \quad (23)$$

4.2.2 Distribution of investors bankruptcy

In the last section we have seen *r/h=0.6* implies a very limited scope for contagion, *r/h=0.9* a large one. These values are thus natural candidates to estimate the no panic and panic case respectively. Regarding the panic threshold, similar to last section we try three values which representing respectively low, moderate, and high tendency for short-term investors to panic, for which panic is triggered for initial shocks of respectively 0.05, 0.01 and 0.005, normalizing *r=1*. This represents 5%, 1% and 0.5% of normalized total quantity of each asset. [16]

Figure 9 to 11 summarizes our findings.

In figure 9, the extreme selling required to trigger panic is very unlikely to happen when $n>2$, so that the figure resembles the non-panic *r/h=0.6* case. Extreme failure, particularly the all fail outcome, very unlikely. On the other hand, when investors only hold their own asset the possibility of a panic exists, which may yield a considerable number of failures, with non trivial odds of as much as 80% of investors going under.

The intermediate case brings new light to our results. Low levels of diversification which were an attractive option without panic now seem particularly harmful. The reason is that such levels are not efficient enough in

[15] Using the conditional variance decomposition formula:
$V(\varepsilon_t) = [V(\triangle\mathbf{P_t}/\forall\triangle q_i :|\triangle q_i| < k*) + (E(\triangle\mathbf{P_t}/\forall\triangle q_i :|\triangle q_i| < k*))^2]P(\forall\triangle q_i :|\triangle q_i| < k*)$
$+[(V(\triangle\mathbf{P_t}/\exists\triangle q_i :|\triangle q_i| > k*) + E(\triangle\mathbf{P_t}/\exists\triangle q_i :|\triangle q_i| > k*))^2]P(\forall\triangle q_i :|\triangle q_i| > k*) - [E(\triangle\mathbf{P_t})]^2$
Since $E(\triangle\mathbf{P_t}) = 0$ and $E(\triangle\mathbf{P_t}/\forall\triangle q_i : \triangle q_i < k*) = 0$, since the distribution of $\triangle Q$ is symmetric.$\triangle\mathbf{P_t}$
$V(\triangle\mathbf{P_t}) = \alpha\sigma_F^2(\frac{hr*}{n^2})^{2t}P(VD_T)^{2t}V(VD_T)^{2t}P + (1-\alpha)\sigma_F^2(\frac{hr}{n^2})^{2t}P(VD_T)^{2t}V(VD_T)^{2t}P$
It follows that $V(\sum_t \triangle\mathbf{P_t}) = \alpha\sigma_F^2 PD_{tot}^* VD_{tot}^* P + (1-\alpha)\sigma_F^2 PD_{tot}VD_{tot}P$
[16] These values may appear low but one should remember our time unit is a "day".

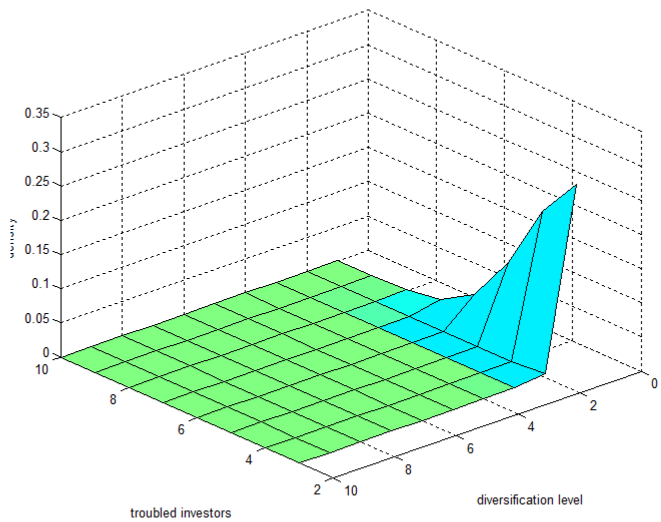

Figure 9: distribution of number of bankruptcies with low panic

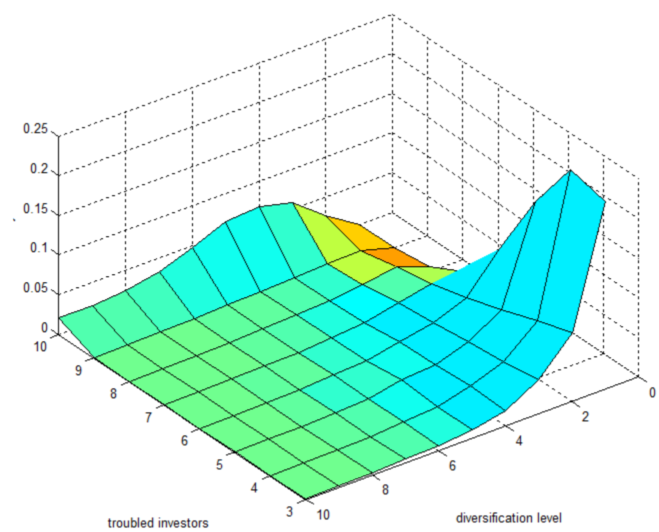

Figure 10: distribution of number of bankruptcies with moderate panic

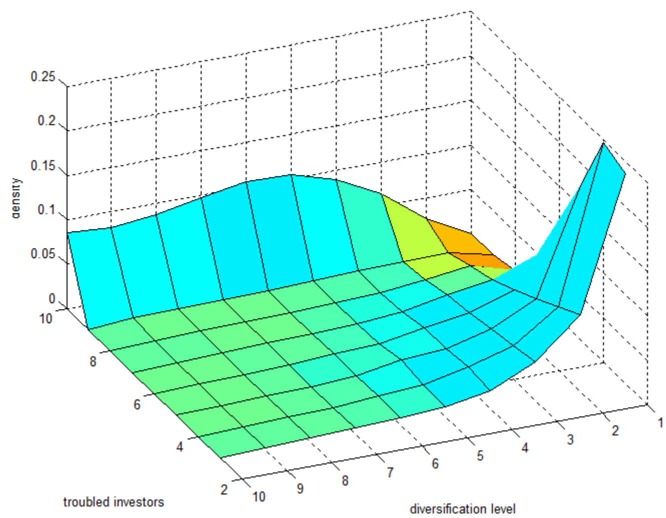

Figure 11: distribution of number of bankruptcies with high panic

smoothing the wealth shocks faced by portfolio holder, but they provide linkages through which shocks may spread across assets. The likelihood of the all-fail outcome is maximized at *n=4*, but the odds that $\eta = 9$ and $\eta = 8$ are also significantly above zero, particularly when *n=2* or *n=3*.

In figure 10 the cumulative probabilities of failure seem to converge towards the *r/h=0.9* case. Yet even with this very low tolerance to fluctuations, the perfectly diversified situation is no longer the most dangerous situation, for any level of failure. The likelihood of the all-fail outcome is maximized at *n=5*, still an intermediary level.

Figures 12 on desirability confirm that introducing panic, higher levels of diversification are no longer the least desirable option, even with steeply increasing costs from mass failure. Low to intermediary levels of diversification are very undesirable in a high contagion context.

5 Extensions

5.1 Patterns of contagion

We generate the stochastic vector of fundamental shocks, and look at the evolution of prices across time and assets.

In the baseline regime we constrain the sum of all asset shocks to be in the bottom 5% of the distribution, ie -1.645 standard deviations away from the mean. Figure 13 shows the evolution of asset prices with *N=10* for levels of diversification 1, 3, 6, and 9, in the baseline scenario. For a given stochastic vector realization, each line represents an the evolution of an asset.

Consistent with our finding on investor bankruptcies, as n increases the price evolutions are more and more homogeneous, as obvious from the *n=6* and *n=9* cases. *n=1* represents the perfectly undiversifed situation, in which the fall on asset i depends only the initial shock on i. *n=3* shows that some of the assets which were rising

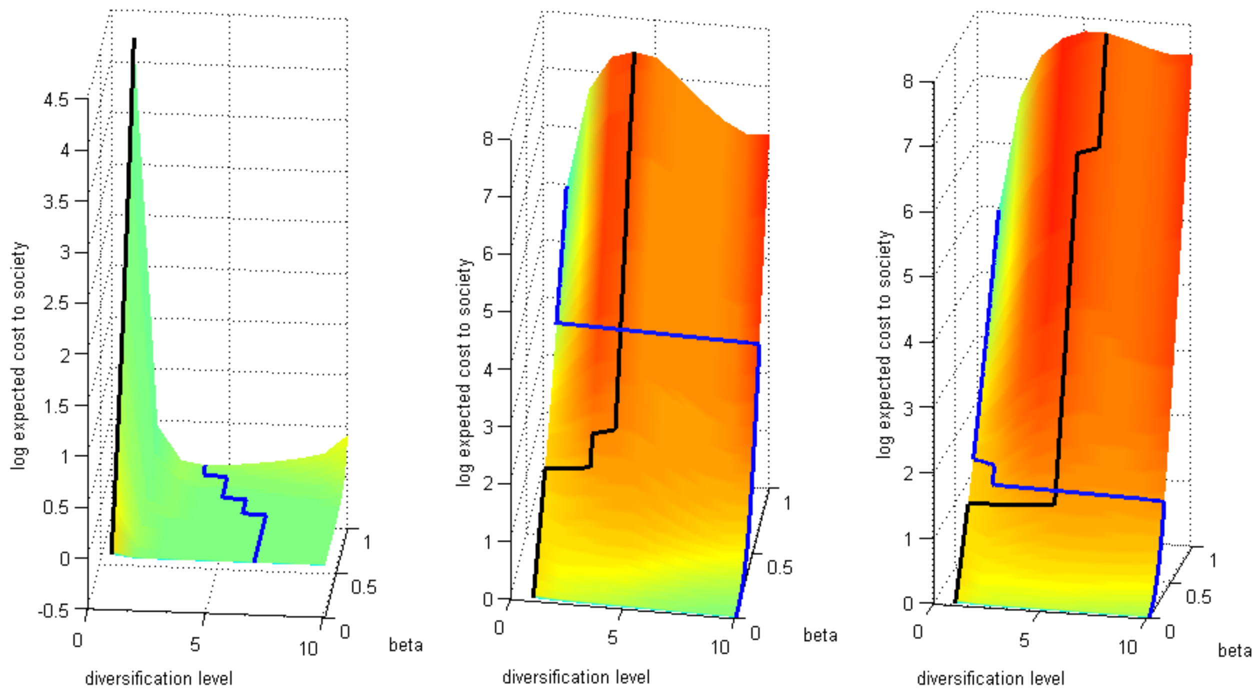

Figure 12: desirability of diversification in the panic regime, with low, moderate, and high scope for panic

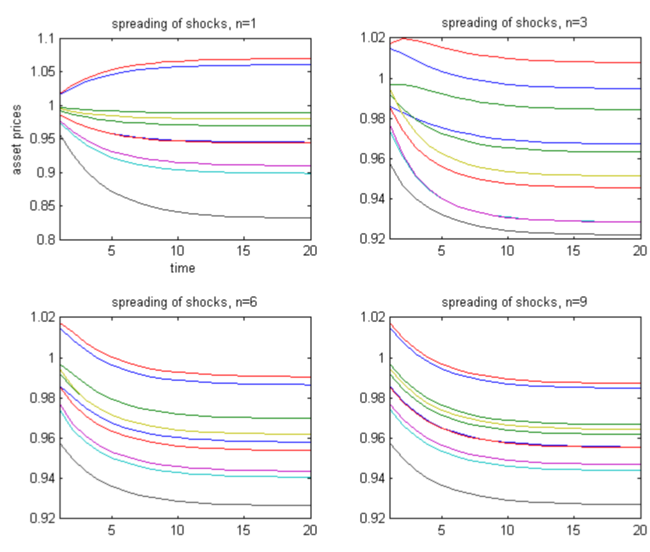

Figure 13: contagion, baseline scenario

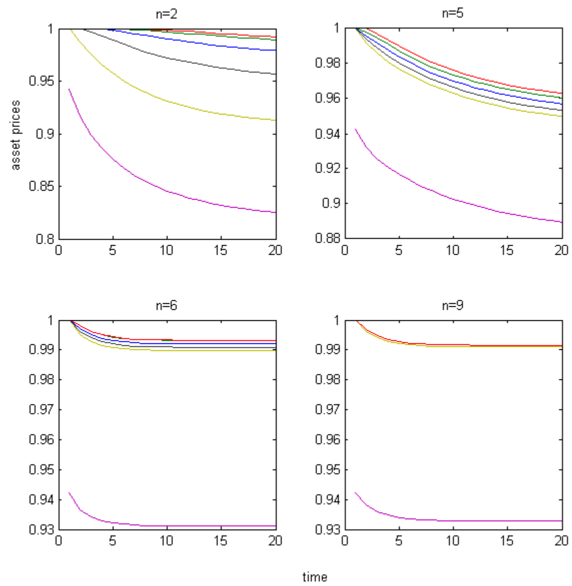

Figure 14: contagion with panic

now fall, sometimes in a pronounced manner, due the downwards pressure from their neighbors. The variance across assets remains quite high nonetheless.

We use a different method for the panic regime, shocking one asset only to see how it propagates. The value of the shock corresponds to the bottom 1% of the distribution of fundamental shocks, that is -2.32 standard deviations away from the mean. We look at diversification levels 2, 5, 6 and 9.

When n=*2* we see a large impact on the shocked asset but also on its neighbors, who lose up to 9% of their value. The case n=*5* cushions this loss across assets, whose losses are distributed around a significant 5%. This is in line with our discussion on covariances between assets, in which we highlighted the fact that two assets may have a higher covariance when they are not directly linked than when they are. In n=*6* the losses for the non-directly shocked asset are much lower. The reason is that at n=*6* panic is no longer triggered, as sales do not exceed the threshold, here 0.01, so that the discount on sales decreases and r/h returns to its *0.6* non-panic level. n=*9* exhibits the same pattern, only further spreading losses over non-directly shocked asset, to the point price falls are nearly equal.

5.2 Alternative network

In our set-up the network we have specified is not efficient from the perspective of portfolio investors, as it increases the covariances between assets of a given portfolio. For instance in our network, when n=3, I_6 holds assets 6, 7, and 8, assets 6 and 7 will be linked through the fact that a shock on 6 lowers the wealth of I_5 and I_6, leading them both to sell asset 7. Imagine now that the pattern of asset holdings is such that I_6 holds 6, 7 , *and 9.* A shock on 6 now leads I_3 to sell assets 3, 4 , 6, I_5 to sell 5, 6, 8, and I_6 to sell 6, 7 ,9. Besides asset 6, no asset has been sold more than once, so that the covariance between 6 and 7 falls. I_6 is safer.

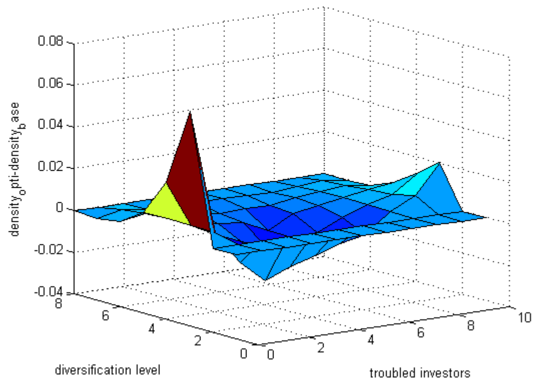

Figure 15: contagion, baseline scenario

We observe the impact of letting portfolio investors move to an "optimal" financial network, in the sense that each investor I it minimizes his connections with any other investor I'. We do so by generating the TT' matrix with all possible networks, taking the square of its elements, and choosing the network for which the sum of these squares is minimized. Figure 15 shows the difference in the probability attached to each number of bankruptcy between this "optimal" network and our baseline one.

Diversification in counterparts has an ambiguous impact on systemic risk, particularly looking at intermediate levels. The individual risk reduction effect kicks in significantly, leading all number of failures between 2 and 8 to be less likely in an optimal network, and the all survive outcome to be much more likely. On the other hand, moving to an optimal network increases the likelihood of the all fail event by a significant 0.02. This comes from the fact that investors now sell assets which are further away in the financial network, spreading shocks more quickly across the markets. Note also that the difference dies out with a large level of diversification as both networks converge when n increases.

Swapping networks thus involves weighing up the same costs and benefits that diversification yields: higher contagion costs versus individually sounder benefits. Looking at desirability we find that for values of $\beta < 0.1$, the degree to which cost is exponential, the optimal level of diversification is higher in an optimal network. Past this level both levels move perfectly together. In words, when mass failure is relatively harmless the individual soundness clearly dominates and the optimal network is preferred. When it mass failures is expensive, the network does not matter, because the cost of mass failure is too high to bear, regardless of the slight difference in probabilities attached to this outcome.

What if we account for panic? Figure 16 shows that this reproduces the same pattern, only more pronounced. The economy is safer for diversification levels up to 8, while the increase in the odds of an extreme failure event have been divided by 2. One could thus conclude that a network free of home bias is marginally preferable in our baseline scenario, and more so accounting for panics. From a model-architecture point of view, this shows that the findings so far are relatively robust to a change network.

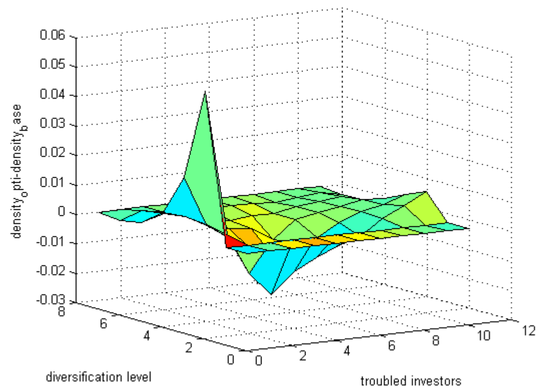

Figure 16: contagion with panic

6 Conclusion

This paper uses a new bottom-up approach to the study of systemic and asset covariances in line with the emerging endogenous risk literature. This allows for a richer analysis of the impact of diversification on systemic risk, for any possible levels and number of defaults, using the properties of circulant matrices. We find that in a rational equilibrium with regulative constraints, diversification increases the probability of mass failure but decreases that of all other non-zero failure outcomes. In a context of strong selling constraints and weak demand, high levels of diversification become undesirable from society's perspective, considering the exponential cost to society of financial failures.

However a global conclusion on the link between diversification and systemic risk edges on a deeper question: is contagion driven primarily by human instincts or by hard-wired features of the financial markets? Indeed as soon introduce heuristics by making the strength of the demand for assets contingent on the severity of the initial shock, a new desirable feature of diversification appears. Lower fluctuations to investors wealth brings lower selling movements, minimizing the possibility of panics. In this context low levels of diversification become particularly harmful, for they create connections between investors without going far enough in minimizing individual risk and the scope for panic. This is more true in the presence of home biases which create niches of risk in the network.

Let us look at the subprime crisis under this light. Credit backed assets were in fact much more closely related than expected by the banks which held them. As these correlations were high, banks were in essence holding a single credit backed asset, their actual diversification level was low. This led the wealth shocks stemming from adverse movements on ABS to be large, which in turn triggered panic through increased counterpart risk and rising risk aversion. According to our set-up a lower level of diversification would have been preferable, for risk would not have spread, and a higher diversification would have the first-best allocation, as banks would have been able to digest the losses from ABS markets without triggering panic.

With respect to policy, an implication of the paper is that the current trend of a more macroprudential regulatory approach may not be optimal when it comes to common asset holding risk, for two reasons. First, in this case the macro and micro approaches appear partly antithetic, for higher interconnectedness and lower probability of default move in opposite directions. Second, as highlighted by the financial stability board (2009), when assets have been subject to a severe shock the failure of any institutions constitutes a systemic event, for in this case the key is to maintain confidence. We thus agree with the current approach of the Basel Committee to leave asset interconnectedness out of the assessment of systemic importance.

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